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A METHOD OF ELICITATION OF INTERVAL TYPE-2 MEMBERSHIP FUNCTIONS AND ITS APPLICATION

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Abstract. *Interval type-2 membership functions (IT2MFs) generalize type-1 membership functions and exhibit capabilities to represent and describe uncertainty. The membership functions' types of IT2MFs could exhibit different forms and are usually elicited from experts or extracted from empirical data. This study establishes a new procedure for generating interval type-2 membership functions (AFSITMFs) utilizing axiomatic fuzzy sets. AFSITMFs are determined by obtaining an interval-valued variance, fixed mean value of observed data, and the given weight information of linguistic terms. In AFSITMFs, randomness derived from the observed data and imprecision originating from the human subjective uncertainty are aggregated in a unified framework. Moreover, an AFSITMF-based classification algorithm is designed by inducing understandable fuzzy descriptions, and some experiments are conducted on publicly available data. The comparative analysis underlines that AFSITMFs-based classification methods can characterize the classification results with semantics.*

Keywords: Interval type-2 fuzzy sets, Membership function, Semantic interpretation, Axiomatic fuzzy sets (AFS)

1. **Introduction.** Data description provides a strategy for expressing a human understandable summarization of data. A more comprehensive description of a collection of numeric data comes not only in the form of some numeric descriptors (mean, median, etc.) but also calls for capturing the data's semantics. Fuzzy models with linguistic terms have significantly contributed to understandable knowledge being extracted from data. Different types of fuzzy set-based methods have been introduced to reflect the underlying rules residing with data, which involve interval sets, type-2 fuzzy sets, intuitionistic fuzzy sets, bipolar fuzzy sets, shadowed sets, etc.

The concept of type-2 fuzzy sets (T2 FSs) was coined by Zadeh in 1975 [1]. T2 FSs have been advocated to capture uncertainties in real-world applications especially control systems [2]. The advances in T2 FSs have resulted in the emergence of new approaches for solving uncertainty problems. However, the effectiveness of T2 FSs in modeling uncertainty comes at the expense of a significant computing overhead for embedding T2 FSs. Thus, interval type-2 fuzzy sets (IT2 FSs) are formed, in which the secondary memberships are equal to 1. IT2 FSs have been reported intensively in the realm of image processing, control systems, decision making, and pattern recognition [3-8].

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Although the theory of type-2 fuzzy sets provides a solution to enhance control systems' performance, the effectiveness depends on their membership functions. Type-2 membership functions are also required to be determined as an initial phase of fuzzy systems applications. The footprint of uncertainty (FOU) of an IT2 FS makes it own more design degrees of freedom than a type-1 fuzzy set [9]. Thus, it is essential to rationally design T2MFs and IT2MFs for the specific context. A detailed review of interval type-2 membership functions can be found in [10]. Recently, data-driven techniques offer an attractive way to determine T2MFs and IT2MFs, in which the strategy of learning and self-tuning parameters is a crucial way to achieve high performance in fuzzy control systems [11]. Data-driven approaches can take full advantage of the probability distribution of collected data. A Gaussian interval type-2 fuzzy neural network was designed by employing a genetic learning algorithm to optimize the parameters [12]. Hosseini et al. established a scheme to learn and tune Gaussian interval type-2 membership functions (IT2MFs) from training data by combining genetic algorithms and cross-validation techniques [6].

This study aims to offer a new data-driven method of generating interval type-2 membership functions from the perspective of combining probability theory and fuzzy theory and also design a classification algorithm with sound accuracy and underlying semantics. The main contributions of this paper are highlighted as follows. 1) A method of generating membership function considering the weight information given by experts and the probability density of observation data is provided. 2) A new classification algorithm is designed to offer sound accuracy and semantics results.

The rest of this paper is organized as follows. Section 2 introduces the AFS and coherence membership functions of fuzzy terms. Section 3 proposes how to construct new IT2 membership functions. Then simulation example is provided to show the performance of the classification algorithm in Section 4. Finally, the conclusions are given in Section 5.

2. **Related Studies.**

AFS framework establishes membership functions and corresponding logic operations that are directly derived from the original data. In this section, we introduce the related concepts of AFS theory.

2.1. **AFS algebras.**

In [13, 14], Liu built a family of AFS algebras on information systems, in which each feature is scaled into different fuzzy concepts with corresponding semantic explanations. Certain logical combinations of fuzzy concepts can characterize the user preferences and object's description with sound semantic description. For the sake of simplicity, the following is introduced to illustrate EI algebras from the viewpoint of semantic cognitive processes.

As mentioned above, any fuzzy concept can be formulated as $\sum_{t\in T}\left(\prod_{m\in A_t} m\right)$, where $A_t \subseteq M$, T denotes the indexing collection, and M is a non-empty set. Then all fuzzy concepts compose the set EM^* as follows.

$$EM^* = \left\{ \sum_{t\in T}\left(\prod_{m\in A_t} m\right) \middle| A_t \subseteq M, t\in T, T \text{ is a predefined indexing collection} \right\}. \quad (1)$$

Two different logical expressions may share the same semantics. At this point, Liu introduced the quotient set EM^*/R by introducing semantic equivalence relation R on EM^* [15]. In addition, to characterize semantic operations "or" and "and", Liu developed two binary operations $\vee$ and $\wedge$ on EM^*/R and further put forward EI algebra $(EM^*/R, \vee, \wedge)$.

Definition 2.1. [16] *For any two elements* $\sum_{s\in S}\left(\prod_{m\in A_s} m\right), \sum_{t\in T}\left(\prod_{m\in B_t} m\right) \in EM^*$*, the binary relation* R *between them is introduced below.* $\left(\sum_{s\in S}\left(\prod_{m\in A_s} m\right)\right) R \left(\sum_{t\in T}\left(\prod_{m\in B_t} m\right)\right) \iff$ *(i)* $\forall A_s$ $(s\in T)$, $\exists B_h$ $(h\in T)$ *such that* $A_s \supseteq B_h$*; (ii)* $\forall B_t$ $(t\in T)$, $\exists A_k$ $(k\in S)$*, such that* $B_t \supseteq A_k$*.*

That means R is an equivalence relation. The quotient set EM^*/R is denoted by EM, and each element of EM is a collection of all equivalence classes with the same semantics.

Theorem 2.1. [15] $(EM, \vee, \wedge)$ *forms a completely distributive lattice under the binary compositions* $\vee$ *and* $\wedge$ *defined as follows: for any* $\sum_{s\in S}\left(\prod_{m\in A_s} m\right), \sum_{t\in T}\left(\prod_{m\in B_t} m\right) \in EM$,

$$\sum_{s\in S}\left(\prod_{m\in A_s} m\right) \vee \sum_{t\in T}\left(\prod_{m\in B_t} m\right) = \sum_{u\in S\sqcup T}\left(\prod_{m\in C_u} m\right), \tag{2}$$

$$\sum_{s\in S}\left(\prod_{m\in A_s} m\right) \wedge \sum_{t\in T}\left(\prod_{m\in B_t} m\right) = \sum_{s\in S, t\in T}\left(\prod_{m\in A_s\cup B_t} m\right), \tag{3}$$

where $u \in S \sqcup T$ *means that* $C_u = A_u$ *if* $u \in S$, *and* $C_u = B_u$ *if* $u \in T$.

The finest semantics description derived from M is denoted as $\prod_{m\in M} m$, and the coarsest one is denoted as $m_1 + m_2 + \cdots + m_{|M|}$ from the view of the underlying semantics.

2.2. **Coherence membership functions of fuzzy terms.** Probability and fuzzy theories are complementary rather than competitive [17, 18], and their connection contributes to practical problem-solving. Inspired by this, Liu and Pedrycz established a method of generating coherence membership functions by combining the data distribution and the fuzzy elements [19].

Definition 2.2. [16, 19] *Let* $(\Omega, \mathcal{F}, \mathcal{P})$ *be a probability measure space,* M *be the collection of simple concepts on* Ω, $\mathcal{F}$ *be all set of Borel sets in* Ω, *and* $\rho_m(y)$ *be the weighting function of* y *in regard to the simple concept* m. *The coherence membership function of fuzzy term* $\xi = \sum_{s\in S}\left(\prod_{m\in A_s} m\right) \in EM$ *can be determined as follows.*

$$\mu_\xi(x) = \sup_{s\in S} \prod_{m\in A_s} \frac{\int_{A_s^\tau(x)} \rho_m(y) d\mathcal{P}(y)}{\int_\Omega \rho_m(y) d\mathcal{P}(y)}, \quad \forall x \in \Omega, \tag{4}$$

where $\Omega = R^k$, $\mathcal{P}(y)$ *is a* k*-normal distribution and* $A_s^\tau(x) = \{y \in \Omega \mid x \succeq_m y \text{ for any } m \in A_s\}$.

In coherence membership functions, the weighting function $\rho_m(y)$ describes the subjective preference with regard to the simple concept m, which also offers a dominance order of different objects on m. The weighting function $\rho_m : \Omega \to [0, +\infty)$ satisfies the following conditions 1) for any $y \in \Omega$, $\rho_m(y) = 0 \Leftrightarrow y \not\succeq_m y$, i.e., y does not belong to m at all; 2) $\rho_m(y) \geq \rho_m(z) \Leftrightarrow y \succeq_m z$, for any $y, z \in \Omega$ [19]. $\rho_m(y)$ can take different ways to reflect subjective imprecision, and its determination depends on the semantics setting of m and the data distribution of the feature that is associated with m. For instance, let m be a simple concept supporting the explanation: "*close to* c_m" and m' be the negation concept of m: "*far from* c_m", whose Gaussian weighting functions can be defined as follows [16].

$$\rho_m(y) = e^{-d_m(f(y)-c_m)^2}, \tag{5}$$

$$\rho_{m'}(y) = 1 - \rho_m(y), \tag{6}$$

where c_m denotes the prototype of the fuzzy concept m, d_m is the parameter that reflects the variance of samples associated with the simple concept m. $f(y)$ represents the value of y with respect to the feature associated with the simple concept m.

3. **Interval Type-2 Membership Functions Developed via Axiomatic Fuzzy Sets.** In (4), the weight function of a simple concept can be influenced by the human perception of the individual's observed data [16]. The different individual's observed data sets can result in the emergence of various membership functions of the same fuzzy concept. Thus, it is necessary to extend (4) to type-2 fuzzy membership functions based on

multiple individual's observed sets. In what follows, we explore a method of determining interval type-2 membership functions.

Usually, there exist two ways of generating interval type-2 membership functions within probability space [20, 21], which cover 1) consider a fixed mean value μ with uncertain standard deviation $\sigma \in [\sigma_1, \sigma_2]$; 2) take account of a fixed standard deviation σ with uncertain mean $\mu \in [\mu_1, \mu_2]$. For instance, two different versions of Gaussian IT2 MFs are shown in Figure 1. Notice that $\mu = 2.5$ and $\sigma = [0.5, 0.75]$ are shown in the left figure, whereas $\mu = [2.5, 3]$ and $\sigma = 0.5$ in the right figure. This study only focuses on eliciting interval type-2 coherence membership functions involving an interval-value spread $[\sigma_1, \sigma_2]$ and a fixed mean value μ.

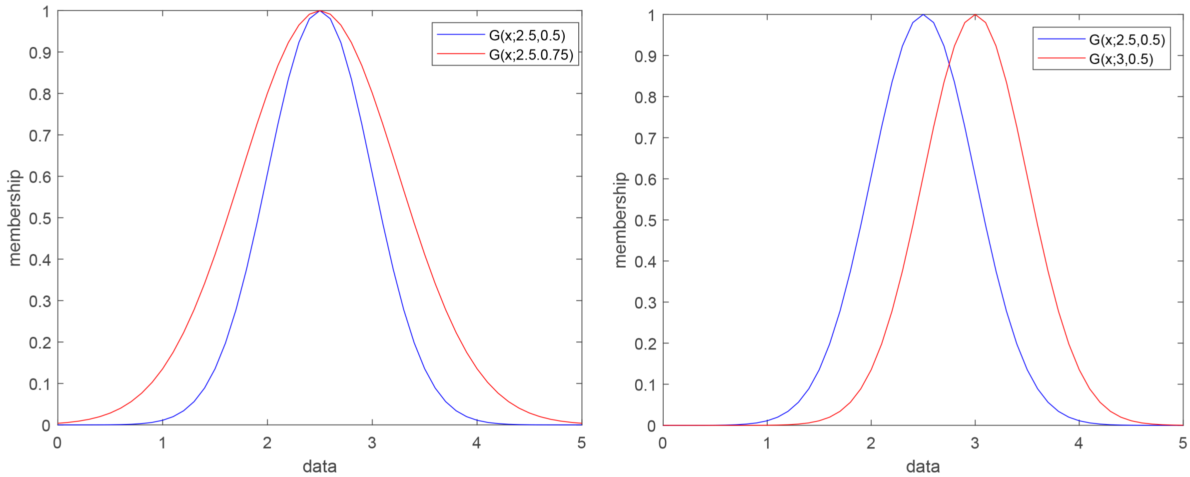

FIGURE 1. Examples of interval type-2 membership functions

3.1. **Constructing new IT2 membership functions.** In what follows, some individual's observed data sets are obtained by using the sampling technique, for which membership functions are established by considering different individual's weight functions and multiple data distribution information. For any simple concept m, an interval-valued spread $[\sigma_1, \sigma_2]$ and a fixed mean value μ_r are obtained based on different observed data sets. The IT2 membership function (AFSITMF) $\tilde{\tilde{\mu}}_\xi(x)$ $\left(\xi = \sum_{s \in S} \left(\prod_{m \in A_s} m\right) \in EM\right)$ can be expressed as follows.

$$\tilde{\tilde{\mu}}_\xi = \left[\widetilde{\mu_\xi^1}, \widetilde{\mu_\xi^2}\right] = \left[\widetilde{\mu_\xi^L}(x; \mu, \sigma_1), \widetilde{\mu_\xi^U}(x; \mu, \sigma_2)\right], \tag{7}$$

$$\widetilde{\mu_\xi^1}(x; \mu, \sigma_1) = \sup_{s \in S} \prod_{m \in A_s} \frac{\int_{A_s^\tau(x)} \rho_m^1(y) d\mathcal{P}(y; \mu, \sigma_1)}{\int_\Omega \rho_m^1(y) d\mathcal{P}(y; \mu, \sigma_1)}, \quad \forall x \in \Omega, \tag{8}$$

$$\widetilde{\mu_\xi^2}(x; \mu, \sigma_2) = \sup_{s \in S} \prod_{m \in A_s} \frac{\int_{A_s^\tau(x)} \rho_m^2(y) d\mathcal{P}(y; \mu, \sigma_2)}{\int_\Omega \rho_m^2(y) d\mathcal{P}(y; \mu, \sigma_2)}, \quad \forall x \in \Omega, \tag{9}$$

where $\mathcal{P}(y; \mu, \sigma_i)$ denotes the probability distribution of y $(i = 1, 2)$, which reflects the range of individual's observed data distribution. $\rho_m^1(y; c_m, \underline{d}_m)$ and $\rho_m^2(y; c_m, \overline{d}_m)$ characterize the subjective uncertainty with respect to the simple concept m within individual's probability spaces. They are defined as follows.

$$\begin{aligned} &\widetilde{\rho_m} = \left[\widetilde{\rho_m^1}(y), \widetilde{\rho_m^2}(y)\right] = \left[\rho_m^1(y; c_m, \underline{d}_m), \rho_m^2(y; c_m, \overline{d}_m)\right], \\ &\rho_m^1(y; c_m, \underline{d}_m) = e^{-\left(\frac{f(y) - c_m}{2\underline{d}_m}\right)^2}, \ \rho_m^2(y; c_m, \overline{d}_m) = e^{-\left(\frac{f(y) - c_m}{2\overline{d}_m}\right)^2}, \\ &\rho_{m'} = 1 - \rho_m, \end{aligned} \tag{10}$$

where c_m, $f(y)$ and m' are identical with (5) and (6). $\underline{d}_m$ and $\overline{d}_m$ are the parameters of reflecting the range of the subjective preferences of fuzzy concept m, and they are set to the variances interval of different samples associated with the user-predefined simple concept m in this study.

In (8) and (9), the terms $\frac{\int_{A_s^\tau(x)} \rho_m^i(y) d\mathcal{P}(y;\mu,\sigma_i)}{\int_\Omega \rho_m^i(y) d\mathcal{P}(y;\mu,\sigma_i)}$ $(i = 1, 2)$ describe the membership grades of x with respect to the simple concept m. The membership functions of the complex concept are composed of the counterparts of single simple concepts by means of max t-conorm 'sup' and product t-norm '$\prod$'. By taking the distribution information of individuals' probability spaces and the semantics of fuzzy concepts into consideration, (8) and (9) deliver potential statistical strategies for extracting type-2 membership functions.

When the cardinality of observed data set $X \subseteq \Omega$ tends to infinity, the following holds [19].

$$\lim_{|X|\to\infty} \frac{\sum_{z\in A_s^\tau(x)} \rho_m^i(z) N_z}{\sum_{z\in X} \rho_m^i(z) N_z} = \frac{\int_{A_s^\tau(x)} \rho_m^i(y) d\mathcal{P}(y;\mu,\sigma_i)}{\int_\Omega \rho_m^i(y) d\mathcal{P}(y;\mu,\sigma_i)}, \quad i = 1, 2, \tag{11}$$

where N_z denotes the number of the sample $z \in A_s^\tau(x)$. By using (11), the membership functions with continuous form can be approximately calculated by discrete approach instead of directly dealing with the high dimensional integral. Moreover, in problem-solving, we can also employ Gaussian functions to approximate these AFSITMFs.

3.2. **An example of forming interval type-2 membership functions via AFS.** The implementation procedure of the proposed method is illustrated by considering a simple example. The Iris data set is represented by a 150 × 4 matrix $X = (x_{ij})_{150\times 4}$, whose elements come from three categories $\{C_1, C_2, C_3\}$. Each sample is described by four features $\{f_1, f_2, f_3, f_4\}$. By taking ten times replacement sampling with a sampling rate of 60% being drawn from X, the values of the interval-value standard variances are obtained.

Let c_{ij} and $\left[\underline{d}_{ij}, \overline{d}_{ij}\right]$ be the mean and the interval-value standard variance of the samples in the class C_i with respect to the feature f_j $(i = 1, 2, 3,\ j = 1, 2, 3, 4)$, respectively, which are listed in Tables 1 and 2.

Table 1. The mean values of iris data

Class	Mean c_{ij}
C_1	$(5.0060, 3.4180, 1.4640, 0.2440)$
C_2	$(5.9360, 2.7700, 4.2600, 1.3260)$
C_3	$(6.5880, 2.9740, 5.5520, 2.0260)$

Table 2. The interval-value standard variances of iris data

Class	$\left[\underline{d}_{ij}, \overline{d}_{ij}\right]$
C_1	$([0.2722, 0.4404], [0.3210, 0.4181], [0.1584, 0.2015], [0.0810, 0.1249])$
C_2	$([0.4283, 0.5923], [0.2857, 0.3655], [0.3862, 0.5504], [0.1528, 0.2314])$
C_3	$([0.5292, 0.7569], [0.2483, 0.3629], [0.4737, 0.6417], [0.2198, 0.3056])$

To illustrate the process of generating AFSITMF, we employ Iris data [22] to obtain interval type-2 membership functions of 24 fuzzy concepts derived from 4 features, respectively. The collection of simple concepts $M = \{m_{ij} \mid 1 \le i \le 4,\ 1 \le j \le 6\}$ is associated with the feature f_j $(j = 1, 2, 3, 4)$ on X. For the semantics of the simple concepts $m \in M$, refer to [19]. For example, m_{11}: "*the sepal length is near about* c_{11}", m_{12} is the negation of m_{11}; m_{13}: "*the sepal length is near about* c_{12}", m_{14} is the negation of m_{13}; m_{15}: "*the* *sepal length is near about* c_{13}", m_{16} is the negation of m_{15}, where c_{ij} is the samples' mean of class C_j with respect to f_i [16].

To obtain the weight functions of a simple concept by taking advantage of the individual's observed data information, in this example, the interval weight information of simples on $m_{ij} \in M$ is described by using Gaussian weighting functions,

$$\begin{cases} \rho^1_{m_{ij}}(y) = e^{-\left(2\underline{d}_{ik}\right)^{-2}(f_i(y)-c_{ik})^2}, & j = 2k-1, \\ \rho^2_{m_{ij}}(y) = e^{-\left(2\overline{d}_{ik}\right)^{-2}(f_i(y)-c_{ik})^2}, & j = 2k-1, \\ \rho^1_{m_{ij}}(y) = 1 - \rho^2_{m_{i(j-1)}}(y), & j = 2k, \\ \rho^2_{m_{ij}}(y) = 1 - \rho^1_{m_{i(j-1)}}(y), & j = 2k, \end{cases} \tag{12}$$

where $y \in X$, $\left[\underline{d}_{ij}, \overline{d}_{ij}\right]$ is the interval-value standard variance of the samples in class C_k of the feature f_i, $i = 1, 2, 3, 4$, $k = 1, 2, 3$.

Based on the weight functions described in (12) and Definition 2.2, we can obtain the IT2 MF of any simple concept $m \in M$ on the total space. To better approximate the membership functions, the Monte Carlo numerical simulation method is employed to calculate the membership degrees. Taking fuzzy concept m_{11} as an example, we generate the lower membership function of simple concept m_{11} by using (11). The membership degree intervals of samples with respect to m_{11} are also obtained in Figure 2, in which the grade of sepal length 5.5 centimeters belonging to m_{11} is $[0.1266, 0.3]$.

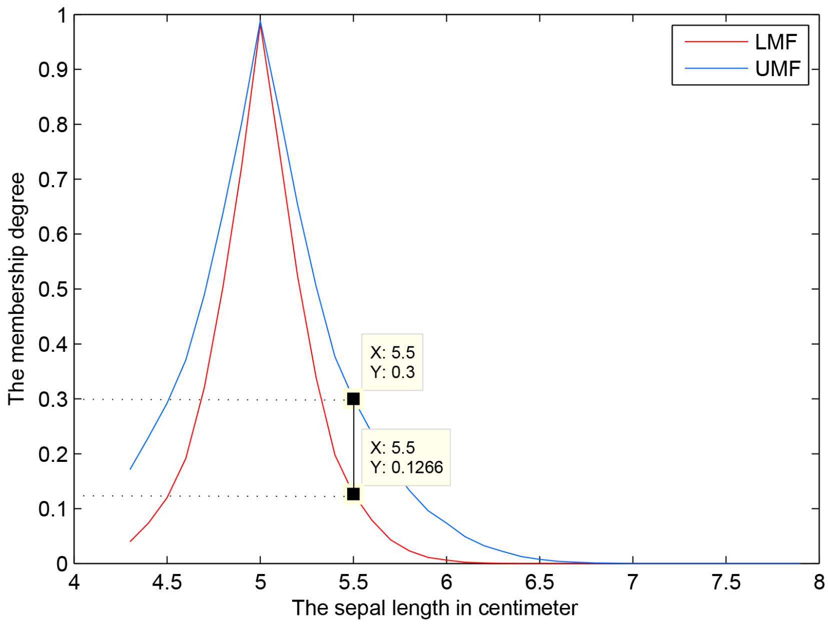

FIGURE 2. An example of membership functions of m_{11}

The designed AFSITMFs are completed in a supervised mode. The proposed method fails to generate the membership function when the data is unlabeled. For this case, we can perform a clustering algorithm in advance.

4. **The Effectiveness of the Established AFSITMFs.** The established AFSITMFs exhibit their underlying semantics, and we examine their effectiveness in solving classification problems. In what follows, we explore a fuzzy classification algorithm to highlight the effectiveness of AFSITMFs, in which the AFSITMFs and semantic rules can be generated from training samples. By finding the description of x_i, we can obtain an overall fuzzy description of each class, by which test samples can be classified based on max-membership principle.

4.1. **Classification method based on AFSITMFs.** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a training set with l different class labels C_j $(j = 1, 2, \ldots, l)$, $M = \{m_1, m_2, \ldots, m_p\}$ be the collection of simple concepts associated with the attributes on X.

Step 1 Determine the description of each object. The best fuzzy description ζ of x_i is formed by finding the complex concept with the maximum distinguished ability from other objects.

- Let small positive numbers $\varepsilon \geq 0$, $\vartheta = \sum_{k=1}^{p} m_k$. Construct $B_{x_i}^{\varepsilon}$ by selecting some potential simple concepts as follows.

$$B_{x_i}^{\varepsilon} = \left\{ m_k \in M \,\middle|\, \mu_{m_k}^{L}(x_i) \geq \mu_{\vartheta}^{L} - \varepsilon, \mu_{m_k}^{U}(x_i) \geq \mu_{\vartheta}^{U} - \varepsilon \right\}, \tag{13}$$

  $B_{x_i}^{\varepsilon}$ is a collection of the simple concept.
- Define the set $\wedge_{x_i}^{\varepsilon,\delta}$ as follows.

$$\wedge_{x_i}^{\varepsilon,\delta} = \left\{ \prod_{m \in A} m \,\middle|\, \mu_{\prod_{m \in A} m}^{L} m(x_i) \geq \delta - \varepsilon, A \subseteq B_{x_i}^{\varepsilon} \right\}. \tag{14}$$

Note 4.1. *We only consider the conjunctions of not more than five simple concepts to alleviate the complexity of the semantics and system overhead.*

Step 2 Given two positive numbers $\varepsilon > 0$, $\delta > 0$. If x_i belongs to the class C_j $(j = 1, 2, \ldots, l)$, the fuzzy description ξ_{x_i} of x_i can be obtained as follows.

$$\xi_{x_i} = \arg\max_{\xi \in F_{\varepsilon}^{\delta}} \left\{ \sum_{x_i \in C_j} \mu_{\xi}^{U}(x_i) \right\}, \tag{15}$$

where

$$E_{\wedge}^{\delta} = \left\{ m \,\middle|\, m \in \wedge_{x_i}^{\varepsilon,\delta}, \forall x_i \in X - C_j, \mu_{\xi}^{U}(x_i) < \delta \right\}, \tag{16}$$

$$F_{\varepsilon}^{\delta} = \left\{ \xi \,\middle|\, \xi \in E_{\wedge}^{\delta}, \forall x_i \in C_j, \mu_{\xi}^{L}(x_i) \geq \delta - \varepsilon \right\}. \tag{17}$$

Step 3 Find the description ζ_{C_j} of the class C_j, where ζ_{C_j} is defined as follows:

$$\zeta_{C_j} = \sum_{x_i \in C_j} \xi_{x_i}. \tag{18}$$

Note 4.2. *For some objects x_i, we cannot find their fuzzy description to meet the condition, which means that the description of x_i is not suitable to be employed to describe this class. The ζ_{C_j} offers a sound semantics of classification rule by the logic operations of simple concepts described in Section 3.2.*

Step 4 For each test sample x_i, $\left[\mu_{\zeta_{C_j}}^{L}(x_i), \mu_{\zeta_{C_j}}^{U}(x_i)\right]$ is the membership degree interval of x_i belonging to class j, $j = 1, 2, 3, \ldots$. The greater the number $\frac{\mu_{\zeta_{C_j}}^{L}(x_i) + \mu_{\zeta_{C_j}}^{U}(x_i)}{2}$ is, the higher the degree of x_i belongs to the class C_j.

4.2. **Experimental studies.** To illustrate the designed classification method, seven classification algorithms in WEKA software, including KNN, Naïve Bayes (NB), Random Forest (RF), AdaBoostM1 (ABM), Classification via Regression (CVR), Logit Boost (LB) and Bagging, are conducted to compare the classification accuracy rate on some UCI data sets. Training data sets in each experiment consist of 60% samples obtained using random sampling, and other samples are considered testing samples. The average classification accuracy rate of 10 times experiments is employed to compare the proposed method with other methods. We have verified the performance of the designed classification method in this study by employing 11 real data sets [22]. The marked numerical values with bold faces mean the current method is better than the other classification algorithms in Table 3. Seven classical classification algorithms obtain better performance on 0, 2, 0, 3, 5, 4,

TABLE 3. Performances of the designed method and other seven classical algorithms

Accuracy	Instances	Features	Class number	AFSITMF	KNN	NB	ABM	Bagging	CVR	LB	RF
Iris	150	4	3	**0.9533**	0.9037	0.8963	0.9483	0.9259	0.7851	0.7037	0.8889
Breast	683	9	2	**0.9667**	0.9608	0.9648	0.9571	0.9619	0.9630	0.9619	0.9633
Wine	178	13	3	0.9657	0.8904	0.9549	0.9178	0.9479	**0.9684**	0.9630	**0.9863**
Glass	214	9	7	**0.6929**	0.5958	0.5544	0.4300	0.6010	0.5440	0.5803	0.5596
Seeds	210	7	3	0.9131	0.9024	0.9107	0.8655	**0.9178**	**0.9452**	**0.9571**	**0.9678**
Ionosphere	351	34	2	**0.9121**	0.8700	0.8285	0.8928	0.9071	0.8642	0.9000	0.9071
Sonar	208	60	2	**0.7243**	0.6631	0.6310	0.6952	0.6791	0.5828	0.6631	0.6364
Balance	625	4	3	0.8469	0.7638	0.7582	0.7627	0.8394	**0.8875**	**0.8702**	0.8245
UK	403	5	4	0.8716	0.8622	**0.8823**	0.5698	**0.9433**	**0.9471**	**0.9257**	**0.9585**
Bloods	748	4	2	0.7701	0.7083	**0.7792**	0.7546	**0.7721**	**0.7875**	**0.7765**	0.7369
Vehicle	946	18	4	**0.6730**	0.6189	0.4704	0.3995	0.5913	0.6609	0.5992	0.6347

and 3 data sets among 11 data sets than the performance of the designed classification method. The accuracy of the designed classification method is higher than that of the other seven classical classification algorithms on Iris, Breast, Glass, Ionosphere, Sonar, and Vehicle data sets.

5. **Conclusions.** In this study, we design a practical method of generating interval type-2 membership functions by combining data distribution information and Gaussian weighting functions, which can be directly derived from observed data and offers new insight into the development of type-2 fuzzy sets within the framework of probability theory. The classification method based on the proposed interval type-2 membership functions embraces knowledge rules inferences with semantics and satisfactory accuracies. The experiments show the effectiveness of the established interval type-2 membership functions. The designed classifier based on the observed samples can also perform the prediction task on the whole data set. The established interval type-2 membership functions-based clustering algorithm is a direction worth further investigating.

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