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UNIFORMLY ASYMPTOTIC STABILITY CRITERION OF SINGULAR TIME-VARYING DELAY SYSTEMS WITH NONLINEAR DISTURBANCES

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Abstract. *For the purpose of studying a class of time-varying delay singular system with nonlinear disturbances, the Lyapunov's second method is employed in this work to determine the uniformly asymptotic stability criterion. Initially, a new Lyapunov Krasovskii Functional (LKF) is presented, followed by the addition of triple integral terms to LKF. Secondly, following derivation, the Bessel-Legendre (B-L) integral inequalities are applied to the integral terms of LKF. By combining the Lyapunov functional approach with the free-weighting matrix method, we obtain the criterion of uniformly asymptotic stability. Notably, compared to the study findings of other integral inequalities, B-L integral inequalities do achieve the uniformly asymptotic stability criterion for singular time-delay systems with nonlinear disturbances. Finally, a numerical example is given to demonstrate the feasibility and superiority of the result obtained.*

Keywords: Singular time-varying delay system, Nonlinear disturbances, Uniform asymptotic stability, Lyapunov-Krasovskii Functional (LKF), The Bessel-Legendre (B-L) integral inequality

1. **Introduction.** Singular systems, also known as implicit systems, descriptor systems, and generalized state-space systems, arise in a range of practical systems, including robotics, the power system and economic systems. The single system model explains the physical system more directly and generically than the standard state space model [1]. Currently, numerous fundamental system theories for developing regular state space systems have been successfully extended to the corresponding singular system theories, such as controllability and observability [2], Lyapunov stability [3], robust stability and stabilization [4], singular time-delay systems [5,6], and optimal control [7].

Because of their extensive applications in many practical systems, a great number of fundamental notions and results in control and system theory based on singular systems have been extended successfully to nonlinear singular systems. For more details on this matter, we refer the reader to [8-15]. Reducing the conservativeness of existing stability criteria remains a core issue in the above references. As we all know, the goal of this problem is to obtain delay-dependent stability conditions. In this study, researchers generally use Jensen integral inequality, Wirtinger integral inequality and auxiliary function integral inequality [8-15] to deal with some integral terms generated by the derivative of L-K function. Compared with Jensen inequality and Wirtinger inequality, B-L inequality enlarges the integral term less, which effectively reduces the conservation and leads to more advantageous. Ding et al. [15] developed criteria for the uniformly asymptotic stability criterion of simple nonlinear singular systems based on the Lyapunov functional approach and the free-weighted matrix method. The criteria significantly decrease the complexity

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of theoretical derivation and computation compared to current literature. Although the stability criteria give a less conservative margin than the strategies described in [8-14], there is still room to delay the reduction of the range boundary's conservatism. Inspired by [15], the authors tried to use B-L inequality to deal with the integral terms generated by the new L-K functions, which has motivated this paper.

Then the primary contribution of this paper is to deal with the problems of uniformly asymptotic stability analysis for nonlinear singular system with time delay. Firstly, we construct a less conservative stability criterion, which guarantees that the system is regular, impulse free and has uniformly asymptotic stability. Secondly, by combining the Lyapunov stability theory and the B-L inequality technique, the conservatism is reduced, and a delay-dependent sufficient condition for nonlinear singular systems with regular, impulse free and uniformly asymptotic stability is proposed. Finally, a concrete example is given to prove the effectiveness and superiority of the proposed method.

2. **Problem Statement and Preliminaries.** Consider the following nonlinear singular system with time-varying delay:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Ff(t,x(t)) + Gg(t,x(t-h(t))) \\ x(t) = \phi(t), \ t \in [-h_2, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state and $\phi(\cdot) \in \mathbb{C}([-h_2, 0], \mathbb{R}^n)$ is a differentiable vector-valued initial continuous function. $\mathbb{R}^n$ is an n-dimensional vector space, defined in the real number field. $E, A, A_d \in \mathbb{R}^{n\times n}$ are known real constant matrices, where E may be singular, $rank(E) = r \le n$, $F \in \mathbb{R}^{n\times n_1}$, $G \in \mathbb{R}^{n\times n_2}$ are the coefficients of the nonlinearities. The nonlinear disturbances $f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n\times n_1}$, $g : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n\times n_2}$ are continuous, $f(t, 0) = 0$, $g(t, 0) = 0$, and satisfy the Lipschitz conditions, that is

$$\|f(t,x(t))\| \le \|u_1 x(t)\|, \ \|g(t,x(t-h(t)))\| \le \|u_2 x(t-h(t))\|. \tag{2}$$

It is assumed that the nonlinear perturbations are bounded in magnitude as where u_i $(i = 1, 2) \in \mathbb{R}^{n_i\times n}$ are constant matrices. And $h(t)$ is a time-varying delay with the differentiable function satisfying $0 \le h_1 \le h(t) \le h_2$, $0 \le \dot{h}(t) \le h_d < 1$, $\forall t \ge 0$, where h_1, h_2, h_d are the bound of time delay, respectively.

The following definitions and lemma are required.

Definition 2.1. [1] *The pair (E, A) is said to be regular if there exists $s \in C$ for which $det(sE - A)$ is not identically zero. The pair (E, A) is said to be impulse free if $deg(det(sE - A)) = rank(E)$.*

Definition 2.2. [15] *The nonlinear singular system (1) is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free.*

Definition 2.3. [15] *The nonlinear singular system (1) is said to be uniformly asymptotically stable if 1) system (1) is regular and impulse free; 2) for any $\epsilon > 0$, there exists a scalar $\delta(\epsilon) > 0$ such that, for any compatible initial conditions $\varphi(t)$ satisfying* $\sup_{-h\le t\le 0} \|\varphi(t)\| \le \delta(\epsilon)$*, the solution $x(t)$ of system (1) satisfies* $\lim_{t\to\infty} x(t) = 0$.

Lemma 2.1. [5] *(B-L integral inequlities) For a symmetric matrix $R > 0$, $R \in \mathbb{R}^{n\times n}$, scalars a and b with $a < b$, a vector function $\dot{x} : [a, b] \to \mathbb{R}^n$, the following integral inequality holds*

$$(b-a)\int_a^b x^T(s)Rx(s)ds \ge {\Lambda_1}^T R\Lambda_1 + 3{\Lambda_2}^T R\Lambda_2 + 5{\Lambda_3}^T R\Lambda_3 + 7{\Lambda_4}^T R\Lambda_4$$

$$\Lambda_1 = \int_a^b x(s)ds, \ \Lambda_2 = -\int_a^b x(s)ds + \frac{2}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta,$$

$$\Lambda_3 = \int_a^b x(s)ds - \frac{6}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta + \frac{12}{(b-a)^2}\int_a^b\int_u^b\int_\theta^b x(s)dsd\theta du,$$

$$\Lambda_4 = -\int_a^b x(s)ds + \frac{12}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta - \frac{60}{(b-a)^2}\int_a^b\int_u^b\int_\theta^b x(s)dsd\theta du + \frac{120}{(b-a)^3}\int_a^b\int_u^b\int_\theta^b\int_\lambda^b x(s)dsd\lambda d\theta du.$$

$$(b-a)^2\int_a^b\int_u^b \dot{x}^T(s)R\dot{x}(s)dsdu \geq 2{\Theta_1}^T R\Theta_1 + 16{\Theta_2}^T R\Theta_2 + 54{\Theta_3}^T R\Theta_3 + 128{\Theta_4}^T R\Theta_4$$

$$\Theta_1 = (b-a)x(b) - \int_a^b x(s)ds,\ \Theta_2 = \frac{b-a}{2}x(b) + \int_a^b x(s)ds - \frac{3}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta,$$

$$\Theta_3 = \frac{b-a}{3}x(b) - \int_a^b x(s)ds + \frac{8}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta - \frac{20}{(b-a)^2}\int_a^b\int_u^b\int_\theta^b x(s)dsd\theta du,$$

$$\Theta_4 = \frac{b-a}{4}x(b) + \int_a^b x(s)ds - \frac{15}{b-a}\int_a^b\int_\theta^b x(s)dsd\theta + \frac{90}{(b-a)^2}\int_a^b\int_u^b\int_\theta^b x(s)dsd\theta du - \frac{210}{(b-a)^3}\int_a^b\int_u^b\int_\theta^b\int_\lambda^b x(s)dsd\lambda d\theta du.$$

3. **Main Results.**

Theorem 3.1. *Given scalars $\varepsilon_1, \varepsilon_2 > 0$ and given delay bound h_1, h_2, h_d, the nonlinear singular system (1) is uniformly asymptotically stable, if there exist positive definite matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3)$, $T_i > 0$ $(i = 1, 2)$, $\begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix} > 0$, $\begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix} > 0$, matrices S, G_1, G_2 with appropriate dimensions and some known matrices R, u_1, u_2, where $R \in \mathbb{R}^{n\times(n-r)}$ is a matrix with full column rank satisfying $E^T R = 0$, such that the following Linear Matrix Inequalities (LMI) holds*

$$\Phi = \begin{bmatrix} \Pi & \Gamma \\ * & \Omega \end{bmatrix} < 0,\ \Omega = \begin{bmatrix} \Omega_1 & O_{4\times 4} \\ * & \Psi_2 \end{bmatrix},\ \Omega_1 = (\Omega_{ij}),\ \Psi_2 = (\Psi_{ij})\ (i, j = 1, 2, 3, 4) \tag{3}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} \\ * & \Pi_{22} & 0 & 0 & \Pi_{25} & \Pi_{26} & \Pi_{27} \\ * & * & \Pi_{33} & \Pi_{34} & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 \\ * & * & * & * & * & * & \Pi_{77} \end{bmatrix} < 0 \tag{4}$$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{35} & \Gamma_{36} & \Gamma_{37} & \Gamma_{38} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} & \Gamma_{45} & \Gamma_{46} & \Gamma_{47} & \Gamma_{48} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$\Pi_{11} = E^T PA + A^T PE + Q_1 + Q_2 + Q_3 + {h_2}^2 W_1 + {h_{12}}^2 Z_1 - 16E^T W_3 E + SR^T A + A^T RS^T + \varepsilon_1 u_1^T u_1 + G_1^T A + A^T G_1 - 10{h_2}^2 E^T T_1 E,$$

$$\Pi_{12} = {h_2}^2 W_2 + {h_{12}}^2 Z_2 + A^T G_2 - G_1^T,\ \Pi_{14} = -4E^T W_3 E,$$

$$\Pi_{15} = E^T PA_d + SR^T A_d + G_1^T A_d,\ \Pi_{16} = E^T PF + SR^T F + {G_1}^T F,$$

$$\Pi_{17} = E^T PG + SR^T G + G_1^T G,\ \Pi_{22} = -G_2 - G_2^T + {h_2}^2 W_3 + {h_{12}}^2 Z_3 + \frac{{h_2}^4}{4} T_1 + \frac{{h_{12}}^4}{4} T_2,$$
$$\Pi_{25} = G_2^T A_d,\ \Pi_{26} = G_2^T F,\ \Pi_{27} = G_2^T G,\ \Pi_{33} = -Q_1 - 16E^T Z_3 E - 10{h_{12}}^2 E^T T_2 E,$$
$$\Pi_{34} = -4E^T Z_3 E,\ \Pi_{44} = -Q_3 - 16E^T W_3 E - 16E^T Z_3 E,$$
$$\Pi_{55} = -\left(1 - h_d\right) Q_2 + \varepsilon_2 u_2^T u_2,\ \Pi_{66} = -\varepsilon_1 I,\ \Pi_{77} = -\varepsilon_2 I,$$
$$\Omega_{11} = -16W_1 - \frac{120}{h_2} W_2 E - \frac{120}{h_2} E^T W_2^T - 100E^T T_1 E - \frac{1200}{{h_2}^2} E^T W_3 E,$$
$$\Omega_{12} = \frac{120}{h_2} W_1 + \frac{1200}{{h_2}^2} E^T W_2^T + \frac{480}{{h_2}^2} W_2 E + \frac{5400}{{h_2}^3} E^T W_3 E + \frac{1200}{h_2} E^T T_1 E,$$
$$\Omega_{13} = -\frac{480}{{h_2}^2} W_1 - \frac{5400}{{h_2}^3} E^T W_2^T - \frac{840}{{h_2}^3} W_2 E - \frac{10080}{{h_2}^4} E^T W_3 E - \frac{6300}{{h_2}^2} E^T T_1 E,$$
$$\Omega_{14} = \frac{840}{{h_2}^3} W_1 + \frac{10080}{{h_2}^4} E^T W_2^T + \frac{13440}{{h_2}^3} E^T T_1 E,$$
$$\Omega_{22} = -\frac{1200}{{h_2}^2} W_1 - \frac{5400}{{h_2}^3} W_2 E - \frac{5400}{{h_2}^3} E^T W_2^T - \frac{25920}{{h_2}^4} E^T W_3 E - \frac{16200}{{h_2}^2} E^T T_1 E,$$
$$\Omega_{23} = \frac{5400}{{h_2}^3} W_1 + \frac{90720}{{h_2}^3} E^T T_1 E + \frac{10080}{{h_2}^4} W_2 E + \frac{25920}{{h_2}^4} E^T W_2^T + \frac{50400}{{h_2}^5} E^T W_3 E,$$
$$\Omega_{24} = -\frac{10080}{{h_2}^4} W_1 - \frac{50400}{{h_2}^5} E^T W_2^T - \frac{201600}{{h_2}^4} E^T T_1 E,$$
$$\Omega_{33} = -\frac{25920}{{h_2}^4} W_1 - \frac{50400}{{h_2}^5} E^T W_2^T - \frac{50400}{{h_2}^5} W_2 E - \frac{100800}{{h_2}^6} E^T W_3 E - \frac{529200}{{h_2}^4} E^T T_1 E,$$
$$\Omega_{34} = \frac{50400}{{h_2}^5} W_1 + \frac{100800}{{h_2}^6} E^T W_2^T + \frac{1209600}{{h_2}^5} E^T T_1 E,$$
$$\Omega_{44} = -\frac{100800}{{h_2}^6} W_1 - \frac{2822400}{{h_2}^6} E^T T_1 E,$$
$$\Psi_{11} = -16Z_1 - \frac{120}{h_{12}} E^T Z_2^T - \frac{120}{h_{12}} Z_2 E - \frac{1200}{{h_{12}}^2} E^T Z_3 E - 100E^T T_2 E,$$
$$\Psi_{12} = \frac{120}{h_{12}} Z_1 + \frac{1200}{{h_{12}}^2} E^T Z_2^T + \frac{480}{{h_{12}}^2} Z_2 E + \frac{5400}{{h_{12}}^3} E^T Z_3 E + \frac{1200}{h_{12}} E^T T_2 E,$$
$$\Psi_{13} = -\frac{480}{{h_{12}}^2} Z_1 - \frac{5400}{{h_{12}}^3} E^T Z_2^T - \frac{840}{{h_{12}}^3} Z_2 E - \frac{10080}{{h_{12}}^4} E^T Z_3 E - \frac{6300}{{h_{12}}^2} E^T T_2 E,$$
$$\Psi_{14} = \frac{13440}{{h_{12}}^3} E^T T_2 E + \frac{840}{{h_{12}}^3} Z_1 + \frac{10080}{{h_{12}}^4} E^T Z_2^T,$$
$$\Psi_{22} = -\frac{16200}{{h_{12}}^2} E^T T_2 E - \frac{1200}{{h_{12}}^2} Z_1 - \frac{5400}{{h_{12}}^3} Z_2 E - \frac{5400}{{h_{12}}^3} E^T Z_2^T - \frac{25920}{{h_{12}}^4} E^T Z_3 E,$$
$$\Psi_{23} = \frac{90720}{{h_{12}}^3} E^T T_2 E + \frac{5400}{{h_{12}}^3} Z_1 + \frac{10080}{{h_{12}}^4} Z_2 E + \frac{25920}{{h_{12}}^4} E^T Z_2^T + \frac{50400}{{h_{12}}^5} E^T Z_3 E,$$
$$\Psi_{24} = -\frac{10080}{{h_{12}}^4} Z_1 - \frac{50400}{{h_{12}}^5} E^T Z_2^T - \frac{201600}{{h_{12}}^4} E^T T_2 E,$$
$$\Psi_{33} = -\frac{529200}{{h_{12}}^4} E^T T_2 E - \frac{25920}{{h_{12}}^4} Z_1 - \frac{50400}{{h_{12}}^5} E^T Z_2^T - \frac{50400}{{h_{12}}^5} Z_2 E - \frac{100800}{{h_{12}}^6} E^T Z_3 E,$$
$$\Psi_{34} = \frac{1209600}{{h_{12}}^5} E^T T_2 E + \frac{50400}{{h_{12}}^5} Z_1 + \frac{100800}{{h_{12}}^6} E^T Z_2^T,$$
$$\Psi_{44} = -\frac{100800}{{h_{12}}^6} Z_1 - \frac{2822400}{{h_{12}}^6} E^T T_2 E,$$
$$\Gamma_{11} = -10h_2 E^T T_1 E + 4E^T W_2^T + \frac{60}{h_2} E^T W_3 E,$$

$$\Gamma_{12} = -\frac{60}{h_2}E^TW_2^T - \frac{360}{{h_2}^2}E^TW_3E + 180E^TT_1E,$$
$$\Gamma_{13} = \frac{360}{{h_2}^2}E^TW_2^T + \frac{840}{{h_2}^3}E^TW_3E - \frac{1260}{h_2}E^TT_1E,\ \Gamma_{14} = -\frac{840}{{h_2}^3}E^TW_2^T + \frac{3360}{{h_2}^2}E^TT_1E,$$
$$\Gamma_{35} = -10h_{12}E^TT_2E + 4E^TZ_2^T + \frac{60}{h_{12}}E^TZ_3E,$$
$$\Gamma_{36} = 180E^TT_2E - \frac{60}{h_{12}}E^TZ_2^T - \frac{360}{{h_{12}}^2}E^TZ_3E,$$
$$\Gamma_{37} = -\frac{1260}{h_{12}}E^TT_2E + \frac{360}{{h_{12}}^2}E^TZ_2^T + \frac{840}{{h_{12}}^3}E^TZ_3E,\ \Gamma_{38} = -\frac{840}{{h_{12}}^3}E^TZ_2^T + \frac{3360}{{h_{12}}^2}E^TT_2E,$$
$$\Gamma_{41} = 16E^TW_2^T + \frac{120}{h_2}E^TW_3E,\ \Gamma_{45} = \frac{120}{h_{12}}E^TZ_3E + 16E^TZ_2^T,$$
$$\Gamma_{42} = -\frac{120}{h_2}E^TW_2^T - \frac{480}{{h_2}^2}E^TW_3E,\ \Gamma_{43} = \frac{480}{{h_2}^2}E^TW_2^T + \frac{840}{{h_2}^3}E^TW_3E,$$
$$\Gamma_{44} = -\frac{840}{{h_2}^3}E^TW_2^T,\ \Gamma_{46} = -\frac{120}{h_{12}}E^TZ_2^T - \frac{480}{{h_{12}}^2}E^TZ_3E,$$
$$\Gamma_{47} = \frac{480}{{h_{12}}^2}E^TZ_2^T + \frac{840}{{h_{12}}^3}E^TZ_3E,\ \Gamma_{48} = -\frac{840}{{h_{12}}^3}E^TZ_2^T,$$
$$e_i = [0_{n\times(i-1)n}, I_n, 0_{n\times(15-i)n}],\ i = 1, 2, \ldots, 15;\ h_{12} = h_2 - h_1.$$

Proof: The proof of this theorem is divided into two parts. The first part deals with the regularity and impulse-free properties and the second part treats the uniformly asymptotic stability property of the studied class of systems. First of all, we show that system (1) is regular and impulse free. According to (3) and Schur complement lemma, $\Pi < 0$.

Let $V_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}$. Pre- and post-multiplying (4) by V_1 and V_1^T, yields

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{15} \\ * & \Pi_{22} & \Pi_{25} \\ * & * & \Pi_{55} \end{bmatrix} - {h_2}^2\begin{bmatrix} W_1 & W_2 & 0 \\ * & W_3 & 0 \\ * & * & 0 \end{bmatrix} - {h_{12}}^2\begin{bmatrix} Z_1 & Z_2 & 0 \\ * & Z_3 & 0 \\ * & * & 0 \end{bmatrix}$$
$$- \begin{bmatrix} Q_1 + Q_2 + Q_3 + \varepsilon_1 u_1^Tu_1 & 0 & 0 \\ * & \frac{{h_2}^4}{4}T_1 + \frac{{h_{12}}^4}{4}T_2 & 0 \\ * & * & \varepsilon_2 u_2^Tu_2 \end{bmatrix} < 0. \quad (6)$$

Let $V_2 = \begin{bmatrix} I & A^T & 0 \\ 0 & {A_d}^T & I \end{bmatrix}$. Pre- and post-multiplying (7) by V_2 and V_2^T, respectively, yields $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0$, $\Sigma_{11} = E^TPA + A^TPE - 16E^TW_3E + SR^TA + A^TRS^T - 10{h_2}^2E^TT_1E$, $\Sigma_{12} = E^TPA_d + SR^TA_d$, $\Sigma_{22} = -(1-h_d)Q_2$.

Since $rankE = r \le n$, there must exist two invertible matrices $\tilde{M}, \tilde{N} \in \mathbb{R}^{n\times n}$ such that

$$\tilde{E} = \tilde{M}E\tilde{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{A} = \tilde{M}A\tilde{N} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \quad (7)$$
$$\tilde{S} = \tilde{N}^TS = \begin{bmatrix} \tilde{S}_{11} \\ \tilde{S}_{21} \end{bmatrix},\ \tilde{A}_d = \tilde{M}P\tilde{N} = \begin{bmatrix} \tilde{A}_{d11} & \tilde{A}_{d12} \\ \tilde{A}_{d21} & \tilde{A}_{d22} \end{bmatrix},\ \tilde{P} = \tilde{M}^{-T}P\tilde{M}^{-1} = \begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix}, \quad (8)$$
$$\tilde{W}_3 = \tilde{M}^{-T}W_3\tilde{M}^{-1} = \begin{bmatrix} \tilde{W}_{311} & \tilde{W}_{312} \\ \tilde{W}_{321} & \tilde{W}_{322} \end{bmatrix}, \tilde{T}_1 = \tilde{M}^{-T}T_1\tilde{M}^{-1} = \begin{bmatrix} \tilde{T}_{111} & \tilde{T}_{112} \\ \tilde{T}_{121} & \tilde{T}_{122} \end{bmatrix}, R = \tilde{M}^T\begin{bmatrix} 0 \\ H \end{bmatrix} \quad (9)$$

where $H \in \mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

Pre- and post-multiply Σ_{11} by $\tilde{N}^T$ and $\tilde{N}$. Using the obtained (7)-(9), we can easily formulate the following inequality: $\Xi = \begin{bmatrix} \sharp & \sharp \\ \sharp & \Xi_{22} \end{bmatrix} < 0$, $\Xi_{22} = \tilde{S}_{21} H^T \tilde{A}_{22} + \tilde{A}_{22}{}^T H \tilde{S}_{21}{}^T$. $\sharp$ is irrelevant to the following discussion. It is easy to see that $\Xi_{22} < 0$. Thus, $\tilde{A}_{22}$ is nonsingular. The pair (E, A) is regular and impulse free. This also means that the system (1) is regular and impulse free.

In the following, we will show that the system (1) is uniformly asymptotically stable. We can choose two nonsingular matrices $\bar{M}, \bar{N} \in R^{n\times n}$ such that

$$\bar{E} = \bar{M}E\bar{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ \bar{A} = \bar{M}A\bar{N} = \begin{bmatrix} \bar{A}_r & 0 \\ 0 & I_{n-r} \end{bmatrix},\ \bar{P} = \bar{M}^{-T}P\bar{M}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix},$$
$$\bar{A}_d = \bar{M}A_d\bar{N} = \begin{bmatrix} \bar{A}_{d11} & \bar{A}_{d12} \\ \bar{A}_{d21} & \bar{A}_{d22} \end{bmatrix},\ \bar{S} = \bar{N}^T S = \begin{bmatrix} \bar{S}_{11} \\ \bar{S}_{21} \end{bmatrix},\ R = \bar{M}^T \begin{bmatrix} 0 \\ \bar{H} \end{bmatrix},$$
$$\bar{F} = \bar{M}F = \begin{bmatrix} \bar{F}_{11} \\ \bar{F}_{12} \end{bmatrix},\ \bar{G} = \bar{M}G = \begin{bmatrix} \bar{G}_{11} \\ \bar{G}_{12} \end{bmatrix},\ \bar{G}_1 = \bar{M}^{-T}G_1\bar{N},\ \bar{G}_2 = \bar{M}^{-T}G_1\bar{M}^{-1},$$
$$\bar{W}_1 = \bar{N}^T W_1 \bar{N} = \begin{bmatrix} \bar{W}_{111} & \bar{W}_{112} \\ \bar{W}_{121} & \bar{W}_{122} \end{bmatrix},\ \bar{W}_2 = \bar{N}^T W_2 \bar{M}^{-1} = \begin{bmatrix} \bar{W}_{211} & \bar{W}_{212} \\ \bar{W}_{221} & \bar{W}_{222} \end{bmatrix},$$
$$\bar{W}_3 = \bar{M}^{-T} W_3 \bar{M}^{-1} = \begin{bmatrix} \bar{W}_{311} & \bar{W}_{312} \\ \bar{W}_{321} & \bar{W}_{322} \end{bmatrix},\ \bar{Z}_1 = \bar{N}^T Z_1 \bar{N} = \begin{bmatrix} \bar{Z}_{111} & \bar{Z}_{112} \\ \bar{Z}_{121} & \bar{Z}_{122} \end{bmatrix},$$
$$\bar{Z}_2 = \bar{N}^T Z_2 \bar{M}^{-1} = \begin{bmatrix} \bar{Z}_{211} & \bar{Z}_{212} \\ \bar{Z}_{221} & \bar{Z}_{222} \end{bmatrix},\ \bar{Z}_3 = \bar{M}^{-T} \bar{Z}_3 \bar{M}^{-1} = \begin{bmatrix} \bar{Z}_{311} & \bar{Z}_{312} \\ \bar{Z}_{321} & \bar{Z}_{322} \end{bmatrix},$$
$$\bar{R} = \bar{M}^{-T} R \begin{bmatrix} 0 \\ \bar{H} \end{bmatrix},\ \bar{T}_1 = \bar{M}^{-T} T_1 \bar{M}^{-1} = \begin{bmatrix} \bar{T}_{111} & \bar{T}_{112} \\ \bar{T}_{121} & \bar{T}_{122} \end{bmatrix},\ \bar{T}_2 = \bar{M}^{-T} T_2 \bar{M}^{-1} = \begin{bmatrix} \bar{T}_{211} & \bar{T}_{212} \\ \bar{T}_{221} & \bar{T}_{222} \end{bmatrix}.$$

At first, we define

$$\begin{aligned} \xi(t) &= col\left\{x(t), E\dot{x}(t), x(t-h_1), x(t-h_2), x(t-h(t)), f(t, x(t)), g(t, x(t-h(t))),\right. \\ &\qquad \left.\eta_1(t), \eta_2(t)\right\}, \\ \eta_1(t) &= col\left\{\int_{t-h_2}^t x(s)ds, \int_{t-h_2}^t \int_\theta^t x(s)dsd\theta, \int_{t-h_2}^t \int_u^t \int_\theta^t x(s)dsd\theta du,\right. \\ &\qquad \left.\int_{t-h_2}^t \int_u^t \int_\theta^t \int_\lambda^t x(s)dsd\lambda d\theta du\right\}, \\ \eta_2(t) &= col\left\{\int_{t-h_2}^{t-h_1} x(s)ds, \int_{t-h_2}^{t-h_1} \int_\theta^{t-h_1} x(s)dsd\theta, \int_{t-h_2}^{t-h_1} \int_u^{t-h_1} \int_\theta^{t-h_1} x(s)dsd\theta du,\right. \\ &\qquad \left.\int_{t-h_2}^{t-h_1} \int_u^{t-h_1} \int_\theta^{t-h_1} \int_\lambda^{t-h_1} x(s)dsd\lambda d\theta du\right\}. \end{aligned}$$

col means column vector. Then choose an LKF candidate as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t), \tag{10}$$
$$V_1(x_t) = x^T(t)E^T PEx(t), \tag{11}$$
$$V_2(x_t) = \int_{t-h_1}^t x^T(s)Q_1x(s)ds + \int_{t-h(t)}^t x^T(s)Q_2x(s)ds + \int_{t-h_2}^t x^T(s)Q_3x(s)ds, \tag{12}$$
$$\begin{aligned} V_3(x_t) &= h_2 \int_{-h_2}^0 \int_{t+\theta}^t \begin{bmatrix} x(s) \\ E\dot{x}(s) \end{bmatrix}^T \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix} \begin{bmatrix} x(s) \\ E\dot{x}(s) \end{bmatrix} dsd\theta \\ &\quad + h_{12} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \begin{bmatrix} x(s) \\ E\dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix} \begin{bmatrix} x(s) \\ E\dot{x}(s) \end{bmatrix} dsd\theta, \end{aligned} \tag{13}$$

$$V_4(x_t) = \frac{{h_2}^2}{2}\int_{t-h_2}^{t}\int_{\lambda}^{t}\int_{\theta}^{t}(E\dot{x}(s))^T T_1(E\dot{x}(s))dsd\theta d\lambda, \tag{14}$$

$$V_5(x_t) = \frac{{h_{12}}^2}{2}\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\int_{\theta}^{t}(E\dot{x}(s))^T T_2(E\dot{x}(s))dsd\theta d\lambda. \tag{15}$$

Let

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \dot{V}_4(x_t) + \dot{V}_5(x_t), \tag{16}$$

$$\dot{V}_1(x_t) = 2x^T(t)E^T P(Ax(t) + A_d x(t-h(t)) + Ff(t,x(t)) + Gg(t,x(t-h(t)))), \tag{17}$$

$$\begin{aligned}\dot{V}_2(x_t) \le{}& x^T(t)(Q_1+Q_2+Q_3)x(t) - x^T(t-h_1)Q_1x(t-h_1)\\ &- x^T(t-h(t))(1-h_d)Q_2x(t-h(t)) - x^T(t-h_2)Q_3x(t-h_2),\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{V}_3(x_t) ={}& {h_2}^2\begin{bmatrix}x(t)\\E\dot{x}(t)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}x(t)\\E\dot{x}(t)\end{bmatrix} + {h_{12}}^2\begin{bmatrix}x(t)\\E\dot{x}(t)\end{bmatrix}^T\begin{bmatrix}Z_1 & Z_2\\ * & Z_3\end{bmatrix}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}\\ &- h_2\int_{t-h_2}^{t}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2\\ * & W_3\end{bmatrix}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}ds\\ &- h_{12}\int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Z_1 & Z_2\\ * & Z_3\end{bmatrix}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}ds.\end{aligned} \tag{19}$$

Assume

$$I_1 = \int_{t-h_2}^{t}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}ds,\ I_2 = \int_{t-h_2}^{t}\int_{\theta}^{t}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\theta,\ I_3 = \int_{t-h_2}^{t}\int_{u}^{t}\int_{\theta}^{t}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\theta du,$$

$$I_4 = \int_{t-h_2}^{t}\int_{u}^{t}\int_{\theta}^{t}\int_{\lambda}^{t}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\lambda d\theta du,\ I_5 = \int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}ds,$$

$$I_6 = \int_{t-h_2}^{t-h_1}\int_{\theta}^{t-h_1}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\theta,\ I_7 = \int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}\int_{\theta}^{t-h_1}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\theta du,$$

$$I_8 = \int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}\int_{\theta}^{t-h_1}\int_{\lambda}^{t-h_1}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}dsd\lambda d\theta du.$$

According to Lemma 2.1, we have

$$\begin{aligned}&- h_2\int_{t-h_2}^{t}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}W_1 & W_2E\\ * & E^TW_3E\end{bmatrix}\begin{bmatrix}x(s)\\\dot{x}(s)\end{bmatrix}ds\\ \le{}& -I_1^T\begin{bmatrix}W_1 & W_2E\\ * & E^TW_3E\end{bmatrix}I_1 - 3\left(-I_1+\frac{2}{h_1}I_2\right)^T\begin{bmatrix}W_1 & W_2E\\ * & E^TW_3E\end{bmatrix}\left(-I_1+\frac{2}{h_1}I_2\right)\\ &- 5\left(I_1-\frac{6}{h_1}I_2+\frac{12}{{h_1}^2}I_3\right)^T\begin{bmatrix}W_1 & W_2E\\ * & E^TW_3E\end{bmatrix}\left(I_1-\frac{6}{h_1}I_2+\frac{12}{{h_1}^2}I_3\right)\\ &- 7\left(-I_1+\frac{12}{h_1}I_2-\frac{60}{{h_1}^2}I_3+\frac{120}{{h_1}^3}I_4\right)^T\begin{bmatrix}W_1 & W_2E\\ * & E^TW_3E\end{bmatrix}\left(-I_1+\frac{12}{h_1}I_2-\frac{60}{{h_1}^2}I_3+\frac{120}{{h_1}^3}I_4\right)\\ &- h_{12}\int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Z_1 & Z_2\\ * & Z_3\end{bmatrix}\begin{bmatrix}x(s)\\E\dot{x}(s)\end{bmatrix}ds\\ \le{}& -I_5^T\begin{bmatrix}Z_1 & Z_2E\\ * & E^TZ_3E\end{bmatrix}I_5 - 3\left(-I_5+\frac{2}{h_{12}}I_6\right)^T\begin{bmatrix}Z_1 & Z_2E\\ * & E^TZ_3E\end{bmatrix}\left(-I_5+\frac{2}{h_{12}}I_6\right)\\ &- 5\left(I_5-\frac{6}{h_{12}}I_6+\frac{12}{{h_{12}}^2}I_7\right)^T\begin{bmatrix}Z_1 & Z_2E\\ * & E^TZ_3E\end{bmatrix}\left(I_5-\frac{6}{h_{12}}I_6+\frac{12}{{h_{12}}^2}I_7\right) - 7\left(-I_5+\frac{12}{h_{12}}I_6\right.\\ &\left.-\frac{60}{{h_{12}}^2}I_7+\frac{120}{{h_{12}}^3}I_8\right)^T\begin{bmatrix}Z_1 & Z_2E\\ * & E^TZ_3E\end{bmatrix}\left(-I_5+\frac{12}{h_{12}}I_6-\frac{60}{{h_{12}}^2}I_7+\frac{120}{{h_{12}}^3}I_8\right)\end{aligned}$$

$$\dot{V}_4(x_t) \le \frac{{h_2}^4}{4}\xi^T(t){e_2}^T T_1 e_2\xi(t) - \xi^T(t)\big(\Gamma_1^T E^T T_1 E\Gamma_1 + 8\Gamma_2^T E^T T_1 E\Gamma_2 + 27\Gamma_3^T E^T T_1 E\Gamma_3 + 64\Gamma_4^T E^T T_1 E\Gamma_4\big)\xi(t) \tag{20}$$

$$\dot{V}_5(x_t) \le \frac{{h_{12}}^4}{4}\xi^T(t)e_2^T T_2 e_2\xi(t) - \xi^T(t)\big\{\Gamma_5^T E^T T_2 E\Gamma_5 + 8\Gamma_6^T E^T T_2 E\Gamma_6 + 27\Gamma_7^T E^T T_2 E\Gamma_7 + 64\Gamma_8^T E^T T_2 E\Gamma_8\big\}\xi(t) \tag{21}$$

$$\Gamma_1 = h_2 e_1 - e_8,\ \Gamma_2 = \frac{h_2}{2}e_1 + e_8 - \frac{3}{h_2}e_9,\ \Gamma_3 = \frac{h_2}{3}e_1 - e_8 + \frac{8}{h_2}e_9 - \frac{20}{{h_2}^2}e_{10},$$
$$\Gamma_4 = \frac{h_2}{4}e_1 + e_8 - \frac{15}{h_2}e_9 + \frac{90}{{h_2}^2}e_{10} - \frac{210}{{h_2}^3}e_{11},\ \Gamma_5 = h_{12}e_3 - e_{12},\ \Gamma_6 = \frac{h_{12}}{2}e_3 + e_{12} - \frac{3}{h_{12}}e_{13},$$
$$\Gamma_7 = \frac{h_{12}}{3}e_3 - e_{12} + \frac{8}{h_{12}}e_{13} - \frac{20}{{h_{12}}^2}e_{14},\ \Gamma_8 = \frac{h_{12}}{4}e_3 + e_{12} - \frac{15}{h_{12}}e_{13} + \frac{90}{{h_{12}}^2}e_{14} - \frac{210}{{h_{12}}^3}e_{15}.$$

For nonlinear functions f, g, given $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, we have

$$0 \le -\varepsilon_1 f^T(t,x(t))f(t,x(t)) + \varepsilon_1 x^T(t)u_1^T u_1 x(t) \tag{22}$$
$$0 \le -\varepsilon_2 g^T(t,x(t-h(t)))g(t,x(t-h(t))) + \varepsilon_2 x^T(t-h(t))u_2^T u_2 x(t-h(t)). \tag{23}$$

It is clear that $E^T R = 0$, and then we can deduce that $0 = 2x^T(t)SR^T E\dot{x}(t)$.

Using the free weighted matrix method, we get that

$$0 = 2\left[x^T(t)G_1^T + (E\dot{x}(t))^T {G_2}^T\right] \times \left[-E\dot{x}(t) + Ax(t) + A_d x(t-h(t)) + Ff(t,x(t)) + Gg(t,x(t-h(t)))\right]. \tag{24}$$

Using the derived derivative terms (17)-(21), add the left and right sides of (22)-(24) into $\dot{V}(x_t)$, and we get $\dot{V}(x_t) \le \xi^T(t)\Phi\xi(t) < 0$.

Thus, the nonlinear singular system (1) is stable. Let $\bar{\Upsilon}$=diag$\big[\bar{N}\ \bar{M}^{-1}\ \bar{N}\ \bar{N}\ \bar{N}\ I\ I\ \bar{N}\ \bar{N}\ \bar{N}\ \bar{N}\ \bar{N}\ \bar{N}\ \bar{N}\ \bar{N}\big]$. Pre- and post-multiply Φ by $\bar{\Upsilon}^T$ and $\bar{\Upsilon}$. We can easily formulate the following inequalities: $\bar{\Phi} = \begin{bmatrix} \bar{\Pi} & \bar{\Gamma} \\ \bar{\Gamma}^T & \bar{\Omega} \end{bmatrix} < 0$. Now, letting $\zeta(t) = \bar{N}^{-1}x(t) = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix}$, where $\zeta_1(t) \in \mathbb{R}^r$, $\zeta_2(t) \in \mathbb{R}^{n-r}$, the nonlinear singular system (1) can be written as

$$\bar{E}\dot{\zeta}(t) = \bar{A}\zeta(t) + \bar{A}_d\zeta(t-h(t)) + \bar{F}f(t,\zeta(t)) + \bar{G}g(t,\zeta(t-h(t))). \tag{25}$$

Let $x(t) = \bar{N}\zeta(t)$ in (10) and $x(t)$ is substituted into $\bar{N}\zeta(t)$, we get

$$V(\zeta(t)) = V_1(\zeta(t)) + V_2(\zeta(t)) + V_3(\zeta(t)) + V_4(\zeta(t)) + V_5(\zeta(t)). \tag{26}$$

We can follow the same line as (17)-(24) to get $\dot{V}(\zeta_t) \le \bar{\xi}^T(\zeta_t)\bar{\Phi}\bar{\xi}(\zeta_t) < 0$, where

$$\bar{\xi}(\zeta_t) = col\left\{\zeta(t), E\dot{\zeta}(t), \zeta(t-h_1), \zeta(t-h_2), \zeta(t-h(t)), f(t,\zeta(t)), g(t,\zeta(t-h(t)))\right\}.$$

Then we have $\lambda_{\min}\left(\bar{P}_{11}\right)\|\zeta_1(t)\|^2 - V(\zeta(0)) \le -\lambda_1\int_0^t \|\zeta_1(s)\|^2 ds < 0$, $\lambda_{\min}\left(\bar{P}_{11}\right)\|\zeta_1(t)\|^2 - V(\zeta(0)) \le -\lambda_1\int_0^t \|\zeta_2(s)\|^2 ds < 0$, if $\lambda_1 = -\lambda_{\max}\left(\bar{\Phi}\right) > 0$. Therefore,

$$0 \le \int_0^t \|\zeta_1(s)\|^2 ds \le \frac{1}{\lambda_1}V(\zeta(0)),\ \int_0^t \|\zeta_2(s)\|^2 ds \le \frac{1}{\lambda_1}V(\zeta(0)). \tag{27}$$

From (27), it is easy to obtain that $\lim_{t\to\infty}\zeta_1(t) = 0$, $\lim_{t\to\infty}\zeta_2(t) = 0$. According to $\zeta(t) = \bar{N}^{-1}x(t)$, we can obtain $\lim_{t\to\infty}x_i(t) = 0$ $(i = 1,2)$. Thus, $\lim_{t\to\infty}x(t) = 0$. According to Definition 2.3, the nonlinear singular system (1) is uniformly asymptotically stable. This completes the proof.

Remark 3.1. *Compared with* [15]*, Theorem 3.1 introduces double and triple integral terms in the construction of L-K functions in order to increase the information of time delay, which reduce conservatism.*

Remark 3.2. *Two B-L integral inequalities are applied in order to deal with LKF integral terms in Theorem 3.1, which would obtain a tighter bound. Consequently, the application of B-L integral inequalities helps to reduce the conservatism of the results efficiently.*

4. Numerical Example.

Example 4.1. [15] *Consider the following nonlinear singular time-delay system (1) with*

$$E\dot{x}(t) = Ax(t) + A_d x(t - h(t)) + Ff(t, x(t)) + Gg(t, x(t - h(t)))$$

where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} -1.8760 & -0.0385 \\ -0.3985 & 2.0366 \end{bmatrix}, \ A_d = \begin{bmatrix} 0.7324 & -1.7906 \\ -0.4120 & 0.6635 \end{bmatrix},$$

$$F = \begin{bmatrix} -0.8876 & -0.0805 \\ -0.9591 & -0.7514 \end{bmatrix}, \ G = \begin{bmatrix} 1.1965 & -0.7897 \\ -0.0348 & 0.9169 \end{bmatrix}.$$

Using these data in [15], a simulation program has been written in Matlab. For this system, we are able to find a feasible solution for the set of LMI for any $h_d \in [0, 0.4772]$. In this example, we choose $R = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. Choosing h_1 as in Table 1 and applying Theorem 3.1 with $h_d = 0.4772$, we find the maximum values of h_2 for which the system remains uniformly asymptotically stable, which are listed in Table 1.

Table 1. The maximum allowable delay bounds h_2 for given h_1 in Example 4.1

Method	$h_1 = 0$	$h_1 = 0.1$	$h_1 = 0.3$	$h_1 = 0.5$	$h_1 = 0.7$	$h_1 = 0.9$	$h_1 = 1$
[15]	0.8617	0.9617	1.1617	1.3617	1.5617	1.7677	1.8825
Theorem 3.1	1.7817	3.9617	4.8709	5.8769	6.9821	7.7774	7.3915

Let us compare the results of Theorem 3.1 in this paper with the ones of Theorem 3.1 in [15]. Theorem 3.1 in [15] gives a delay-range-dependent uniformly asymptotic stability criterion for system (1). The paper selected the upper bound of time-varying delay h_1 which is the same as [15]. For given different values of h_1, using Theorem 3.1, the values of h_2 are shown in Table 1. The maximum value of h_2 that makes the system uniformly asymptotically stable in [15] is also listed in Table 1. It is obvious that h_2 obtained by applying Theorem 3.1 is larger than h_2 of [15]. Therefore, the proposed method presented in this paper provides less conservative results than previous results.

5. Conclusions. In this paper, the uniformly asymptotic stability of singular time delay systems with nonlinear disturbances has been investigated. The augmented LKF is established, and the B-L integral inequality and the double B-L integral inequality are used to deal with the integral terms of the derivative of LKF. A numerical example demonstrates that the proposed method is less conservative than those in previous studies. In future, state feedback controller or output feedback controller could be considered in order to control the singular time-delay systems with nonlinear disturbances.

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