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TRACKING CONTROL OF NONLINEAR SYSTEM WITH PARTIALLY AVAILABLE STATES

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Abstract. *The tracking control of nonlinear system with partially available states is investigated. The nonlinear system in this paper can be considered to be composed of the two coupled subsystems: the subsystem with all available state variables and the other subsystem with all unavailable state variables. The dynamics of the two subsystems are represented by vector differential equations, respectively. By using the available state and the given system tracking reference targets, the state feedback tracking controller is designed for the nonlinear system, which can guarantee that the available state and the unavailable state asymptotically track the given reference targets, respectively. In other words, the results in this paper show that when the available state asymptotically tracks a given reference target, the unavailable state can also asymptotically track the other given reference target. Compared with the existing literature, this paper directly utilizes the available state variables to design the tracking controller instead of utilizing any state observers to estimate the unavailable state of the system, which not only maintains the dimension of the system to be constant but also reduces the control operation delay. Finally, a numerical simulation example is given to verify the validity of the results in this paper.*

Keywords: Available state variables, Nonlinear system, Tracking control, Unavailable state variables, State feedback controller

1. **Introduction.** In the field of practical engineering application, some state variables of the controlled system cannot be directly detected due to sensor technology, design requirements or cost saving considerations, for example, the permanent magnet synchronous motor drive system without speed sensor [1, 2], the AC/DC servo system without speed sensor [3, 4], and the winding system without tension sensor [5, 6]. Evidently, the unavailable state variables mean that they cannot be detected and used directly in control scheme, so them will make a direct difficulty on the synthesis of state feedback controller. On the other hand, during system operation, the controller may have some sensor faults in the detection state beyond expectations. In this case, the controlled system can be regarded as the operation under the condition with unavailable state variables, which will also cause the system controller failure, leading to the instability of the whole system [7, 8, 9, 10]. Thinking via the perspective of large system theory [11], we can consider the nonlinear system with available state variables and unavailable state variables as a composite system, which is composed of the two subsystems, one is with the available state variables, and the other is with unavailable state variables. Therefore, how to design the

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controller for the system under the condition that some state variables are unavailable is a problem worth considering.

In order to solve the above problem, the existing literature mainly considers the control design based on system state observers [12, 13, 14, 15], containing the full dimensional state observers [12, 13] and the reduced order observers [14, 15]. For instance, in [12], a full dimensional state observer is designed for linear systems with unavailable disturbances, providing a simple and efficient method for linear systems with unmeasurable disturbances to estimate disturbances, and solves the shortcomings of previous many scholars who must assume that unavailable disturbances satisfy the given constant coefficient differential equation and that all inputs are assumed to be known states to design state observers. In [13], a general nonlinear full-dimensional observer is designed for fuzzy systems with unavailable variables, which is suitable for discrete and continuous systems, and solves the mismatch problem of the system caused by the premise variables of unavailable variables. In [14], when studying distributed output feedback tracking control of nonlinear multi-agent systems, in order to solve the problem of dependence of dynamic nonlinear functions on available state variables of the system, the reduced order observer is used to estimate the unavailable state variables, and the corresponding relationship between network topology and the reduced order observer is established to achieve the consistency of output of leaders and followers in the system. In [15], a reduced order observer of the nonlinear system is designed for the unavailable state vector contained in the system to estimate the unmeasurable state, and the coupling between the reduced order observer and the feedback controller is used to finally complete the design of the system output feedback controller, when studying the global stabilization and stability problem of smooth control output feedback of high-order switched nonlinear systems with uncontrollable unobservable subsystems linearized by Jacobian matrices. The above examples show the control design methods based on the state observers, the essence of which is to design state observers to replace the unavailable state variables with the corresponding estimated state variables, so as to finish the purpose of designing state feedback controllers for the system.

However, the control methods via state observers have some disadvantages [16, 17, 18]. On the one hand, the state observer increases the state dimensions of the whole system (which consists of the original controlled system and state observer), which easily causes the time-delay of the controller operation; on the other hand, owing to the nonlinear and unknown inputs, disturbances and other environment uncertainties, the state observer has a large estimation deviation in actual operation. In order to overcome the above two shortcomings, designing a robust state observer for the system is one of useful methods [19, 20, 21]. However, it is a useful alternative method without the state observer that the control scheme is synthesized only by using the available state variables and the boundedness information involved in available state variables. This alternative method is rare in the existing literature.

Influenced by the above discussion, this paper focuses on the tracking control of composite system without the state observer. Compared with the existing literature, the main contributions and innovations of this paper are as follows. (i) Different from the previous literature, this paper regards the controlled system as the composite system with the two coupled subsystems, one is described by the available state variables, and the other is described by the unavailable state variables. The available state variables are used to synthesize the tracking controller without the state observer, which is rare in the existing literature. (ii) The feedback gains in the synthesized tracking controller are determined mainly by the boundedness information involved in available state variables. This implies that the tracking controller is robust for the unavailable state variables to a certain extent.

The rest of this manuscript is organized as follows. In Section 2, the dynamic differential equations for the composite system are proposed, which is composed of the two subsystems, one is with available state variables, and the other is with unavailable state variables, where the two subsystems are mutually coupled. In addition, the control goal of this paper is presented and the tracking control scheme is also designed. In Section 3, the simulation example is given to demonstrate the validity of control scheme. Finally, the conclusion is given in Section 4.

2. **Model Description and Control Design.** In this paper, we consider the nonlinear composite system with the state vector $x = x(t) = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T \in R^n$, where $x_1 \in R^{n_1}$ is available state vector, and $x_2 \in R^{n_2}$ is unavailable state vector, $n_1 + n_2 = n$. The dynamic differential equations of nonlinear composite system can be described as follows:

$$\dot{x_1} = A_1(x_1, t)x_1 + \Phi_1(x_1, t)x_2 + G_1u_1 \tag{1}$$

$$\dot{x_2} = A_2(x_1, t)x_2 + \Phi_2(x_1, t)x_1 + G_2u_2 \tag{2}$$

where $A_1 = A_1(x_1, t) \in R^{n_1 \times n_1}$, $A_2 = A_2(x_1, t) \in R^{n_2 \times n_2}$, $\Phi_1 = \Phi_1(x_1, t) \in R^{n_1 \times n_2}$, $\Phi_2 = \Phi_2(x_1, t) \in R^{n_2 \times n_1}$ are matrix functions of state vector x_1, $u_1 \in R^{n_1}$, $u_2 \in R^{n_2}$ are control inputs for the subsystem (1) and subsystem (2), respectively, $G_1 \in R^{n_1 \times n_1}$ and $G_2 \in R^{n_2 \times n_2}$ are gain matrices, $n_1 \le n_2$.

Remark 2.1. *The models of linear composite systems with two subsystems can be regarded as the special ones of Equations (1) and (2), if the matrices $A_1 = A_1(x_1, t)$, $A_2 = A_2(x_1, t)$, $\Phi_1 = \Phi_1(x_1, t)$, $\Phi_2 = \Phi_2(x_1, t)$ are constant matrices* [22, 23, 24]. *For some dynamic systems, the auxiliary system models can be represented in form of Equations (1) and (2). This can refer to the simulation example (the double coupled inverted pendulums system) in Section 3.*

Let $A = A(x_1, t) = \textit{block-diag}\begin{bmatrix} A_1(x_1, t) & A_2(x_1, t) \end{bmatrix} \in R^{n \times n}$, $\Phi = \Phi(x_1, t) = \begin{bmatrix} O_{n_1 \times n_1} & \Phi_1(x_1, t) \\ \Phi_2(x_1, t) & O_{n_2 \times n_2} \end{bmatrix} \in R^{n \times n}$, $G = \textit{block-diag}\begin{bmatrix} G_1 & G_2 \end{bmatrix} \in R^{n \times n}$, $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in R^n$. Therefore, by the above symbols, Equations (1) and (2) can be rewritten as follows:

$$\dot{x} = Ax + \Phi x + Gu \tag{3}$$

Assumption 2.1. *Consider the nonlinear composite system with Equations (1) and (2). The state vector x_1 is available and x_2 is unavailable. The matrices $A_1(x_1, t)$, $A_2(x_1, t)$, $\Phi_1(x_1, t)$, $\Phi_2(x_1, t)$ are known and bounded, and the gain matrices G_1 and G_2 are known with inversible matrices G_1^{-1} and G_2^{-1}, respectively.*

Let $x_1^* = x_1^*(t) \in R^{n_1}$ and $x_2^* = x_2^*(t) \in R^{n_2}$ be two given bounded tracking targets of subsystem (1) and subsystem (2), respectively, with the bounded derivatives. Introduce the error vectors $e_1 = e_1(t) = x_1(t) - x_1^*(t)$, $e_2 = e_2(t) = x_2(t) - x_2^*(t)$, and tracking error $e = x - x^* = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$, in which $x^* = \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}$. The following error differential equation can be obtained by using Equation (3).

$$\dot{e} = Ae + \Phi e + \Phi x^* + Ax^* - \dot{x}^* + Gu \tag{4}$$

For a real symmetric matrix D, the inequality $D < 0$ means that the matrix D is negative definition.

Assumption 2.2. *Consider the nonlinear composite system with Equations (1) and (2). The following Inequality (5) is true.*

$$A_2 + A_2^T + \frac{1}{\mu + 1}\left(B^T\Phi_1 + \Phi_1^T B\right) < 0 \tag{5}$$

where μ is an adjustable positive parameter ($\mu > 0$). $B = \begin{bmatrix} I_{n_1} & O_{n_1 \times (n_2 - n_1)} \end{bmatrix} \in R^{n_1 \times n_2}$, the I_{n_1} denotes the n_1-order identity matrix.

It is seen that its transpose matrix $B^T = \begin{bmatrix} I_{n_1} \\ O_{(n_2-n_1)\times n_1} \end{bmatrix}$ satisfies $BB^T = I_{n_1}$, where $O_{p\times q}$ denotes the $p \times q$ zero matrix.

Remark 2.2. *If $A_2+A_2^T$ is the Hurwitz matrix with bounded norm, namely, its maximum eigenvalue $\lambda_{\max}\left(A_2 + A_2^T\right) \le -\gamma$, and γ is a positive real number, λ represents matrix eigenvalue, $\| * \|$ denotes the norm of matrix "$*$".*

Due to $\frac{1}{\mu+1}\lambda_{\max}\left(B^T\Phi_1 + \Phi_1^T B\right) \le \frac{1}{\mu+1}\left\|B^T\Phi_1 + \Phi_1^T B\right\|$, we can select the μ that satisfies the inequality $\frac{1}{\mu+1}\left\|B^T\Phi_1 + \Phi_1^T B\right\| < \varepsilon < \gamma$, further $\frac{1}{\mu+1}\lambda_{\max}\left(B^T\Phi_1 + \Phi_1^T B\right) < \varepsilon$ (ε is any positive real number). Consequently, the eigenvalue of matrix $\Big[A_2 + A_2^T + \frac{1}{\mu+1}\left(B^T\Phi_1 + \Phi_1^T B\right)\Big]$ makes the following Inequality (6) true.

$$\begin{aligned}\lambda\left[A_2 + A_2^T + \frac{1}{\mu+1}\left(B^T\Phi_1 + \Phi_1^T B\right)\right] &\le \lambda_{\max}\left(A_2 + A_2^T\right) + \frac{1}{\mu+1}\lambda_{\max}\left(B^T\Phi_1 + \Phi_1^T B\right) \\ &\le -\gamma + \varepsilon < 0 \end{aligned} \tag{6}$$

The above Inequality (6) implies that Assumption 2.2 is true.

Control goal. Consider the nonlinear composite system with Equations (1) and (2). For the given bounded signal $x^* = \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} \in R^n$ with the bounded derivative, if the state vector $x_2 = x_2(t)$ is unavailable, design the control input $u = u(x_1, x^*, t)$ for the subsystems (1) and (2) such that $e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \xrightarrow{t\to+\infty} 0$ holds, that is to say, the state vectors $x_1(t)$ and $x_2(t)$ can track the given reference goals $x_1^*(t)$ and $x_2^*(t)$, respectively.

Remark 2.3. *According to Assumption 2.2, the matrix $B^TB = \begin{bmatrix} I_{n_1} \\ O_{(n_2-n_1)\times n_1} \end{bmatrix} \begin{bmatrix} I_{n_1} & O_{n_1\times(n_2-n_1)} \end{bmatrix} = \begin{bmatrix} I_{n_1} & O_{n_1\times(n_2-n_1)} \\ O_{(n_2-n_1)\times n_1} & O_{(n_2-n_1)\times(n_2-n_1)} \end{bmatrix}$, then, defining matrix $I_{n_2} = \begin{bmatrix} I_{n_1} & O_{n_1\times(n_2-n_1)} \\ O_{(n_2-n_1)\times n_1} & I_{n_2-n_1} \end{bmatrix}$, hence, it is easy to obtain $[(\mu+1)I_{n_2} - B^TB] = \begin{bmatrix} \mu I_{n_1} & O_{n_1\times(n_2-n_1)} \\ O_{(n_2-n_1)\times n_1} & (\mu+1)I_{n_2-n_1} \end{bmatrix}$, this proves that the matrix $[(\mu+1)I_{n_2} - B^TB]$ is invertible and $[(\mu+1)I_{n_2} - B^TB]^{-1} = \begin{bmatrix} \mu^{-1} I_{n_1} & O_{n_1\times(n_2-n_1)} \\ O_{(n_2-n_1)\times n_1} & \frac{1}{\mu+1}I_{n_2-n_1} \end{bmatrix}$, particularly, if $n_1 = n_2$, then $\left[(\mu+1)I_{n_2} - B^TB\right]^{-1} = \mu^{-1}I_{n_1}$.*

In order to achieve the above control goal, the control input u_1 of subsystem (1) and the control input u_2 of subsystem (2) are proposed, respectively.

$$u_1 = G_1^{-1}\left(-K_{n_1\times n_1}e_1 - \Phi_1 x_2^* - A_1 x_1^* + \dot{x}_1^*\right) \tag{7}$$

$$u_2 = G_2^{-1}\left(-\bar{K}_{n_2\times n_1}e_1 - \Phi_2 x_1^* - A_2 x_2^* + \dot{x}_2^*\right) \tag{8}$$

where the gain matrices $\bar{K}_{n_2\times n_1}$ and $K_{n_1\times n_1}$ of the control inputs are proposed as follows:

$$\begin{aligned}\bar{K}_{n_2\times n_1} = &\begin{bmatrix} \mu^{-1} I_{n_1} & O_{n_1\times(n_2-n_1)} \\ O_{(n_2-n_1)\times n_1} & \dfrac{1}{\mu+1}I_{n_2-n_1} \end{bmatrix}\left\{\Phi_1^T + (\mu+1)\Phi_2 + A_2^T B^T + B^T A_1\right. \\ &\left. - B^T[0.5\delta I_{n_1} + B\Phi_2 + A_1]\right\}\end{aligned} \tag{9}$$

$$K_{n_1\times n_1} = 0.5\delta I_{n_1} + B\Phi_2 + A_1 - B\bar{K}_{n_2\times n_1} \tag{10}$$

where δ is an adjustable positive parameter ($\delta > 0$).

Equations (7) and (8) can be rewritten as follows:

$$u = G^{-1}\left[-\begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix} - \Phi x^* - Ax^* + \dot{x}^*\right] \tag{11}$$

It is easily verified by Equations (9) and (10) that the following equations are true.

$$-\delta I_{n_1} = A_1 - K_{n_1\times n_1} + B\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) + A_1^T - K_{n_1\times n_1}^T + \left(-\bar{K}_{n_2\times n_1}^T + \Phi_2^T\right)B^T \tag{12}$$

$$O_{n_2\times n_1} = \Phi_1^T + A_2^T B^T + B^T\left(A_1 - K_{n_1\times n_1}\right) + (\mu+1)\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) \tag{13}$$

Theorem 2.1. *Consider the nonlinear composite system with Equations (1) and (2), if Assumptions 2.1 and 2.2 are true, for the given bounded signal* $x^* = \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} \in R^n$ *with the bounded derivative, the designed control scheme (7)-(10) can ensure that* $e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{pmatrix} \xrightarrow{t\to+\infty} 0$.

Proof: Consider the block-matrix $P = \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \in R^{n\times n}$. By using Remark 2.3 into the Schur Complement theorem [25], it can be verified that the P is the positive definite matrix. So, $V = V(e) = e^T Pe$ is the positive definite function about the entries of e.

Through Assumptions 2.1 and 2.2 and the control scheme (7)-(10) with (12) and (13), the orbit derivative of $V = V(e)$ along the error system (4) is obtained as follows.

$$\begin{aligned}
\dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\
&= 2e^T P\dot{e} \\
&= 2e^T \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \left\{Ae + \Phi e + \Phi x^* + Ax^* - \dot{x}^* + Gu\right\} \\
&= 2e^T \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \left\{\begin{bmatrix} A_1 & O_{n_1\times n_2} \\ O_{n_2\times n_1} & A_2 \end{bmatrix} e + \begin{pmatrix} \Phi_1 e_2 \\ \Phi_2 e_1 \end{pmatrix} - \begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix}\right. \\
&\quad \left. + \begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix} + \Phi x^* + Ax^* - \dot{x}^* + Gu\right\} \\
&= 2e^T \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \left\{\begin{bmatrix} A_1 & O_{n_1\times n_2} \\ O_{n_2\times n_1} & A_2 \end{bmatrix} e + \begin{bmatrix} -K_{n_1\times n_1} & \Phi_1 \\ -\bar{K}_{n_2\times n_1} + \Phi_2 & O_{n_2\times n_2} \end{bmatrix} e\right. \\
&\quad \left. + \begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix} + \Phi x^* + Ax^* - \dot{x}^* + Gu\right\} \\
&= 2e^T \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \left\{\begin{bmatrix} A_1 - K_{n_1\times n_1} & \Phi_1 \\ -\bar{K}_{n_2\times n_1} + \Phi_2 & A_2 \end{bmatrix} e + \begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix}\right. \\
&\quad \left. + \Phi x^* + Ax^* - \dot{x}^* + Gu\right\} \\
&= 2e^T \begin{bmatrix} I_{n_1} & B \\ B^T & (\mu+1)I_{n_2} \end{bmatrix} \begin{bmatrix} A_1 - K_{n_1\times n_1} & \Phi_1 \\ -\bar{K}_{n_2\times n_1} + \Phi_2 & A_2 \end{bmatrix} e \\
&\quad + 2e^T \left\{\begin{pmatrix} K_{n_1\times n_1}e_1 \\ \bar{K}_{n_2\times n_1}e_1 \end{pmatrix} + \Phi x^* + Ax^* - \dot{x}^* + Gu\right\} \\
&= 2e^T \begin{bmatrix} A_1 - K_{n_1\times n_1} + B\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) & \Phi_1 + BA_2 \\ B^T\left(A_1 - K_{n_1\times n_1}\right) + (\mu+1)\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) & B^T\Phi_1 + (\mu+1)A_2 \end{bmatrix} e
\end{aligned}$$

$$= e^T \begin{bmatrix} A_1 - K_{n_1\times n_1} + B\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) & \Phi_1 + BA_2 \\ B^T\left(A_1 - K_{n_1\times n_1}\right) + (\mu+1)\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right) & B^T\Phi_1 + (\mu+1)A_2 \end{bmatrix} e$$
$$+ e^T \begin{bmatrix} A_1^T - K_{n_1\times n_1}^T + \left(-\bar{K}_{n_2\times n_1}^T + \Phi_2^T\right)B^T & \left(A_1^T - K_{n_1\times n_1}^T\right)B + (\mu+1)\left(-\bar{K}_{n_2\times n_1}^T + \Phi_2^T\right) \\ \Phi_1^T + A_2^T B^T & \Phi_1^T B + (\mu+1)A_2^T \end{bmatrix} e$$
$$= e^T \begin{bmatrix} M_1 + M_1^T & M_2 \\ M_2^T & B^T\Phi_1 + (\mu+1)A_2 + \Phi_1^T B + (\mu+1)A_2^T \end{bmatrix} e$$
$$= e^T \begin{bmatrix} -\delta I_{n_1} & O_{n_1\times n_2} \\ O_{n_2\times n_1} & B^T\Phi_1 + (\mu+1)A_2 + \Phi_1^T B + (\mu+1)A_2^T \end{bmatrix} e \tag{14}$$

where matrices in Equation (14) $M_1 = A_1 - K_{n_1\times n_1} + B\left(-\bar{K}_{n_2\times n_1} + \Phi_2\right)$, $M_2 = \Phi_1 + BA_2 + \left(A_1^T - K_{n_1\times n_1}^T\right)B + (\mu+1)\left(-\bar{K}_{n_2\times n_1}^T + \Phi_2^T\right)$.

By Assumption 2.2, we get that $\dot{V}(e) < 0$ and

$$\Psi = \begin{bmatrix} -\delta I_{n_1} & O_{n_1\times n_2} \\ O_{n_2\times n_1} & B^T\Phi_1 + (\mu+1)A_2 + \Phi_1^T B + (\mu+1)A_2^T \end{bmatrix}$$

is negative definite. Therefore, it can be seen by using Equation (14) that the error system (4) with the control scheme (7)-(10) is asymptotically stable in Lyapunov sense. This means that $e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \stackrel{t\to+\infty}{\longrightarrow} 0$. So, Theorem 2.1 is proved.

Remark 2.4. *To apply Theorem 2.1, the following steps are proposed.*

(I) Determine the known and bounded function matrices $A_1(x_1,t)$, $A_2(x_1,t)$, $\Phi_1(x_1,t)$, $\Phi_2(x_1,t)$ and the known inversible matrices G_1, G_2, in Equations (1) and (2). Verify whether Assumptions 2.1 and 2.2 are true for the above matrices.

(II) Give the bounded and derivatives bounded reference signal targets x_1^* and x_2^* concerning $x_1(t)$ and $x_2(t)$, respectively.

(III) Give the matrix B according to n_1 and n_2, adjustable positive parameter μ in Assumption 2.2.

(IV) Take the above matrices and parameters into the control scheme (7)-(10), which can ensure that tracking error vector $e \stackrel{t\to+\infty}{\longrightarrow} 0$.

3. **Simulation Example.** Consider the double coupled inverted pendulums system (DCIPS) with the following dynamic models [26].

$$I_1\ddot{\varphi}_1 = m_1 g l_1 \varphi_1 + kh^2(\varphi_2 - \varphi_1) - c_1\dot{\varphi}_1 - m_1 l_1 u_{11} - m_1 l_1 \dot{u}_{12} \tag{15}$$
$$I_2\ddot{\varphi}_2 = m_2 g l_2 \varphi_2 + kh^2(\varphi_2 - \varphi_1) - c_2\dot{\varphi}_2 - m_2 l_2 u_{21} - m_2 l_2 \dot{u}_{22} \tag{16}$$

where φ_1 and φ_2 are the swing angles of two inverted pendulums with the masses m_1 and m_2, the damping coefficients are c_1 and c_2, and the lengths are l_1 and l_2, respectively. The spring stiffness coefficient is k and the h is the distance between the spring fixation point and the suspension point of the pendulums. The u_1 and u_2 are control inputs for the DCIPS.

Here, assume that the swing angle φ_1 and its angular speed $\dot{\varphi}_1$ of the first pendulum are available, and the swing angle φ_2 and the angular speed $\dot{\varphi}_2$ of the second pendulum are unavailable.

The control goal in this simulation is to control the swing angles φ_1 and φ_2 to track the given reference targets φ_1^* and φ_2^*, respectively. In order to use the control scheme in this paper for achieving the above control goal in this simulation, the following auxiliary system models with (17) and (18) can be obtained in form of Equations (1) and (2).

Selecting the system state variables $x_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix}$, $x_2 = \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix}$ with $x_{11} = \varphi_1$, $x_{12} = -\alpha_1\varphi_1 + \dot{\varphi}_1 + \frac{m_1 l_1}{I_1}u_{12}$, $x_{21} = \varphi_2$, $x_{22} = -\alpha_2\varphi_2 + \dot{\varphi}_2 + \frac{m_2 l_2}{I_2}u_{22}$ and the control inputs $u_1 = \begin{pmatrix} u_{11} \\ u_{12} \end{pmatrix}$, $u_2 = \begin{pmatrix} u_{21} \\ u_{22} \end{pmatrix}$, where α_1 and α_2 are the adjustable parameters to be chosen, by which and Equations (15) and (16), we can obtain the auxiliary dynamic models of DCIPS as follows:

$$\dot{x}_1 = \begin{bmatrix} \alpha_1 & 1 \\ \dfrac{m_1 g l_1}{I_1} - \dfrac{kh^2}{I_1} - \alpha_1\left(\alpha_1 + \dfrac{c_1}{I_1}\right) & -\left(\alpha_1 + \dfrac{c_1}{I_1}\right) \end{bmatrix} x_1 + \begin{bmatrix} 0 & 0 \\ \dfrac{kh^2}{I_1} & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 & -\dfrac{m_1 l_1}{I_1} \\ -\dfrac{m_1 l_1}{I_1} & \dfrac{m_1 l_1}{I_1}\left(\alpha_1 + \dfrac{c_1}{I_1}\right) \end{bmatrix} u_1 \tag{17}$$

$$\dot{x}_2 = \begin{bmatrix} \alpha_2 & 1 \\ \dfrac{m_2 g l_2}{I_2} - \dfrac{kh^2}{I_2} - \alpha_2\left(\alpha_2 + \dfrac{c_2}{I_2}\right) & -\left(\alpha_2 + \dfrac{c_2}{I_2}\right) \end{bmatrix} x_2 + \begin{bmatrix} 0 & 0 \\ \dfrac{kh^2}{I_2} & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 & -\dfrac{m_2 l_2}{I_2} \\ -\dfrac{m_2 l_2}{I_2} & \dfrac{m_2 l_2}{I_2}\left(\alpha_2 + \dfrac{c_2}{I_2}\right) \end{bmatrix} u_2 \tag{18}$$

For Equations (17) and (18) and the given reference goals $x_1^* = \begin{pmatrix} x_{11}^* \\ x_{12}^* \end{pmatrix}$, $x_2^* = \begin{pmatrix} x_{21}^* \\ x_{22}^* \end{pmatrix}$, the control goal in this simulation is that the swing angles x_{11} and x_{21} track the given reference targets $x_{11}^* = \varphi_1^*(t)$ and $x_{21}^* = \varphi_2^*(t)$, respectively. The other state variables x_{12} and x_{22} are auxiliary variables, which can track the given reference targets $x_{12}^* = x_{12}^*(t)$ and $x_{22}^* = x_{22}^*(t)$, respectively.

In simulation, choose the parameters inspired in [26, 27] as follows: $m_1 = m_2 = 1.0$ kg, $l_1 = l_2 = 0.5$ m, $h = \frac{l_1}{2} = \frac{l_2}{2} = 0.25$ m, $I_1 = m_1 \times l_1^2$, $I_2 = m_2 \times l_2^2$. The spring stiffness coefficient $k = 100$ N/m; the damping coefficients $c_1 = 3.5$, $c_2 = 3.33$.

Comparing (17) and (18) with Equations (1) and (2), the function matrices $A_1(x_1, t)$, $A_2(x_1, t)$, $\Phi_1(x_1, t)$, $\Phi_2(x_1, t)$ and gains G_1, G_2 are obtained as follows:

$$A_1(x_1, t) = \begin{bmatrix} \alpha_1 & 1 \\ \dfrac{m_1 g l_1}{I_1} - \dfrac{kh^2}{I_1} - \alpha_1\left(\alpha_1 + \dfrac{c_1}{I_1}\right) & -\left(\alpha_1 + \dfrac{c_1}{I_1}\right) \end{bmatrix}, \quad \Phi_1(x_1, t) = \begin{bmatrix} 0 & 0 \\ \dfrac{kh^2}{I_1} & 0 \end{bmatrix}$$

$$A_2(x_1, t) = \begin{bmatrix} \alpha_2 & 1 \\ \dfrac{m_2 g l_2}{I_2} - \dfrac{kh^2}{I_2} - \alpha_2\left(\alpha_2 + \dfrac{c_2}{I_2}\right) & -\left(\alpha_2 + \dfrac{c_2}{I_2}\right) \end{bmatrix}, \quad \Phi_2(x_1, t) = \begin{bmatrix} 0 & 0 \\ \dfrac{kh^2}{I_2} & 0 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0 & -\dfrac{m_1 l_1}{I_1} \\ -\dfrac{m_1 l_1}{I_1} & \dfrac{m_1 l_1}{I_1}\left(\alpha_1 + \dfrac{c_1}{I_1}\right) \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0 & -\dfrac{m_2 l_2}{I_2} \\ -\dfrac{m_2 l_2}{I_2} & \dfrac{m_2 l_2}{I_2}\left(\alpha_2 + \dfrac{c_2}{I_2}\right) \end{bmatrix}$$

In addition, it is easily verified that $\frac{c_2}{2I_2} \geq 1 + \sqrt{1 + r}$, $k \geq \frac{m_2 g l_2}{h^2}$ $\left(r = \frac{kh^2}{I_2} - 1 - \frac{m_2 g l_2}{I_2}\right)$ is true. In order to ensure that Assumption 2.2 is true, the adjustable parameter $\alpha_1 < 0$

can be selected arbitrarily and α_2 is chosen to satisfy the following two inequalities:

$$\frac{1}{2}\left[-\frac{c_2}{I_2}+\sqrt{\frac{c_2^2}{I_2^2}-4\left(1+\sqrt{1+r}\right)^2}\right]<\alpha_2<\frac{1}{2}\left[-\frac{c_2}{I_2}+\sqrt{\frac{c_2^2}{I_2^2}-4\left(1-\sqrt{1+r}\right)^2}\right] \quad (19)$$

or

$$-\frac{1}{2}\left[\frac{c_2}{I_2}+\sqrt{\frac{c_2^2}{I_2^2}-4\left(1-\sqrt{1+r}\right)^2}\right]<\alpha_2<-\frac{1}{2}\left[\frac{c_2}{I_2}+\sqrt{\frac{c_2^2}{I_2^2}-4\left(1+\sqrt{1+r}\right)^2}\right] \quad (20)$$

In simulation, select $\alpha_1 = -13.5$, $\alpha_2 = -14.48$, $\mu = 9$. The reference targets of two swing angles $x_{11}^* = \varphi_1^* = \sin(t)$ and $x_{21}^* = \varphi_2^* = \cos(t)$, the reference targets of auxiliary variables $x_{12}^* = \sin(t)$ and $x_{22}^* = \cos(t)$.

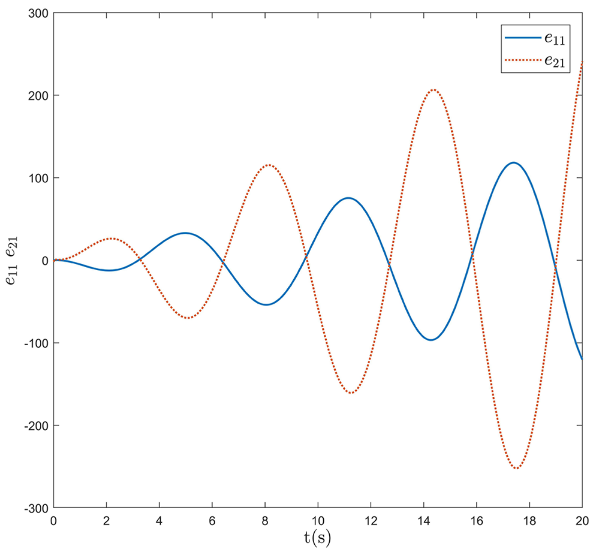
The simulation results are shown in Figures 1-3, in which Figure 3 shows that the comparison results by using controllers via the reduced-order observers in [20, 21] and state feedback controller in this paper, where the comparison results are indicated by the tracking error curves of two swing angles φ_1 and φ_2, as well as two auxiliary variables x_{12} and x_{22}.

Let $e_{11} = x_{11} - x_{11}^* = \varphi_1 - \varphi_1^*$, $e_{21} = x_{21} - x_{21}^* = \varphi_2 - \varphi_2^*$, $e_{12} = x_{12} - x_{12}^*$, $e_{22} = x_{22} - x_{22}^*$.

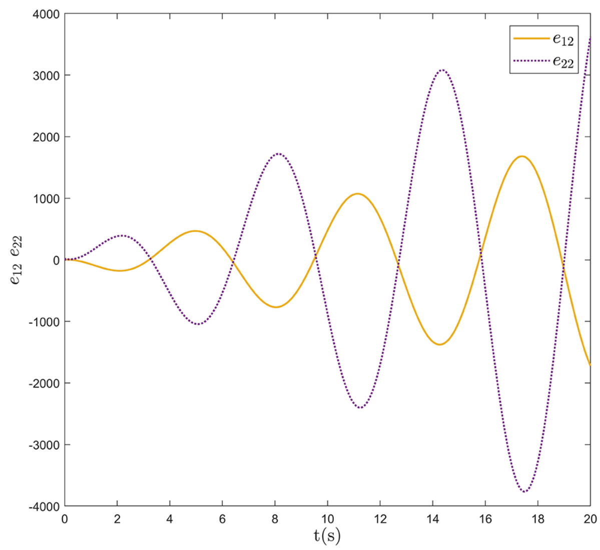
From the simulation figures, the following conclusions can be obtained.

(I) Figure 1 shows that the tracking errors of two swing angles φ_1, φ_2 and two auxiliary variables x_{12}, x_{22} are divergent. This implies that the swing angles φ_1, φ_2 and the auxiliary variables x_{12}, x_{22} cannot track their respective reference targets without the controller in this paper.

(II) Figure 2 shows that the swing angle tracking errors and auxiliary variables tracking errors converge asymptotically to zero, respectively. This implies that the controller in this paper can ensure not only the swing angles φ_1 and φ_2 track asymptotically the given reference goals φ_1^* and φ_2^*, respectively, but also the auxiliary variables x_{12}, x_{22} can track asymptotically the given reference goals x_{12}^* and x_{22}^*, respectively. This verified the validity of the control scheme proposed in this paper.



(a)



(b)

FIGURE 1. The error response curves without controller in this paper: (a) The tracking errors of two swing angles φ_1 and φ_2; (b) the tracking errors of auxiliary variables x_{12} and x_{22}

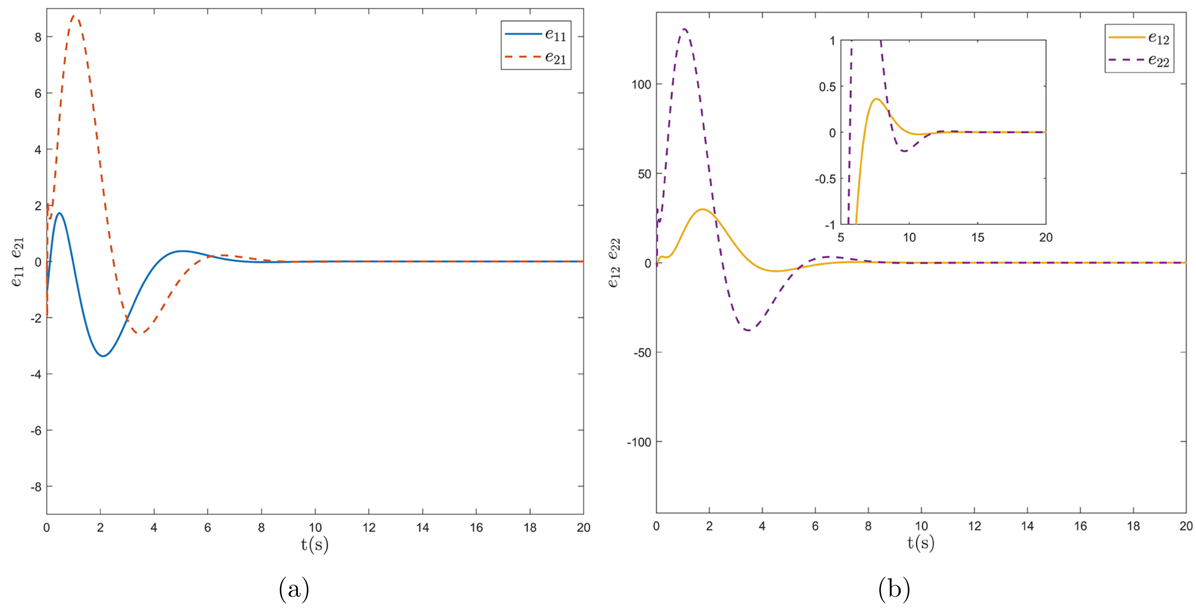


FIGURE 2. The error response curves with controller in this paper: (a) The tracking errors of two swing angles φ_1 and φ_2; (b) the tracking errors of auxiliary variables x_{12} and x_{22}

(III) In Figure 3, (a) and (b) show that the comparison results by using the controllers based on the reduced-order observers in [20, 21] and this paper, respectively. It is easy to observe that with the controller in this paper, the tracking errors of swing angles can quickly and asymptotically converge to zero. However, the tracking errors cannot converge to zero with the controllers in [20, 21]. The above results mean that compared to the observers with the partially unavailable state variables that are not utilized in the tracking control scheme, the controller proposed in this paper is more suitable than ones in [20, 21]. (c) shows that the auxiliary variables x_{12} and x_{22} are bounded. This implies that the auxiliary variables introduced in this paper do not cause the unboundedness in the control process. In addition, from $x_{12} = -\alpha_1\varphi_1 + \dot{\varphi}_1 + \frac{m_1 l_1}{I_1}u_{12}$ and $x_{22} = -\alpha_2\varphi_2 + \dot{\varphi}_2 + \frac{m_2 l_2}{I_2}u_{22}$ with the controllers (7) and (8), it is seen that the boundedness of x_1 and x_2 can guarantee the boundedness of angular velocities of the inverted pendulums.

4. **Conclusion.** In this paper, the dynamic model of nonlinear composite system, which is composed of the available state subsystem and the unavailable state subsystem has been proposed. In the existing literature mainly focused on the tacking control of the nonlinear system with unavailable state variables, designing the state observer to estimate unavailable state is important one of control synthesis methods. Compared with the existing literature, the main advantage of this paper is to synthesize the asymptotic tracking control scheme only by using the available state variables, in which any observers for unavailable state are not used. In simulation example, the auxiliary variables are chosen so that the dynamic model of the double coupled inverted pendulums can be transformed into a standard form in this paper. However, the tracking control of nonlinear systems with unavailable state in this paper is based on the accurate mathematical model of the system, and the system matrices and parameters are known. Hence, the tracking control of nonlinear systems with partially unavailable state variables and uncertainties will be researched in the future work.

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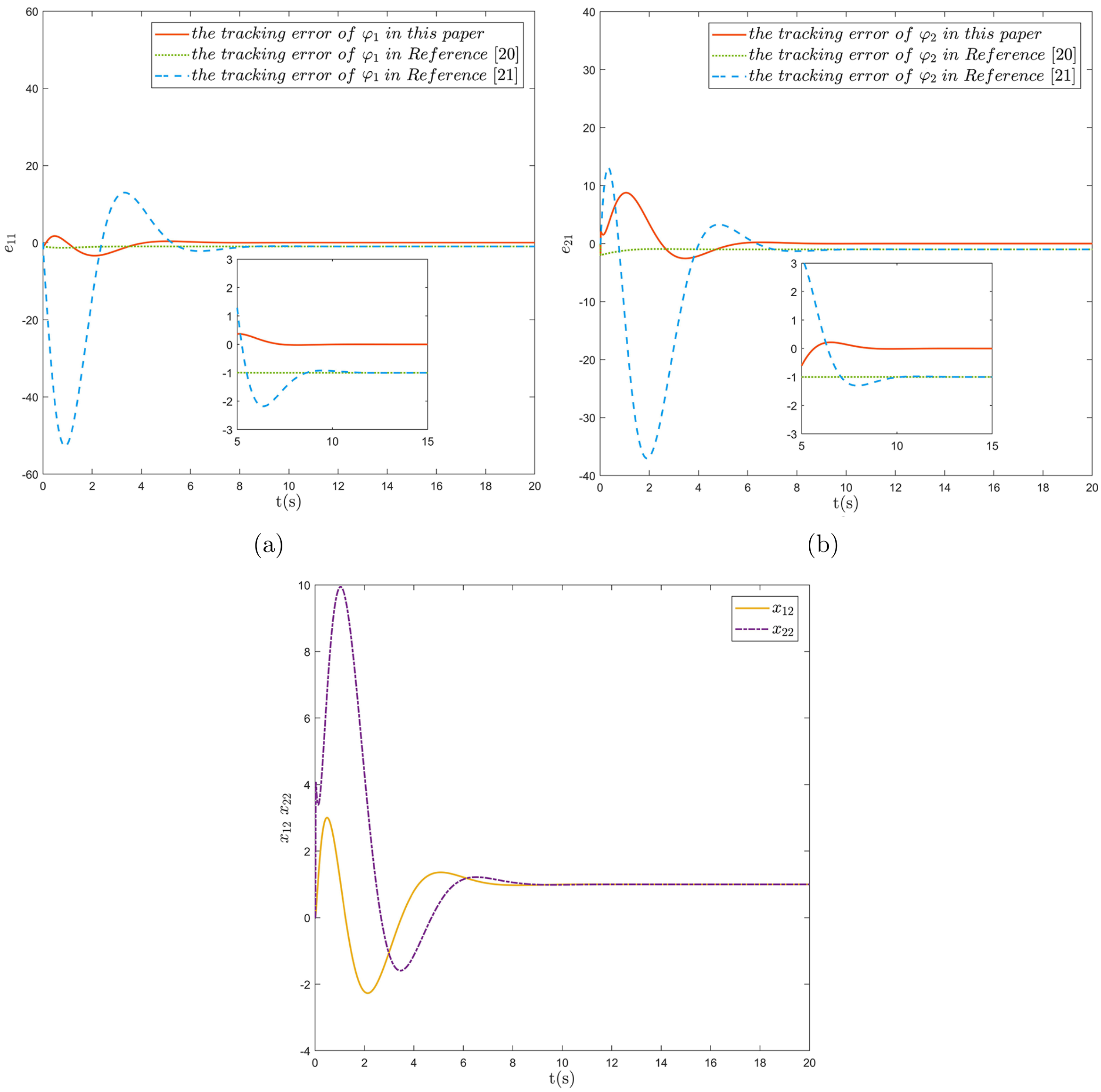

FIGURE 3. The error and auxiliary variable response curves with controller in this paper: (a) The tracking errors of swing angle φ_1 with the controllers in [20, 21] and in this paper; (b) the tracking errors of swing angle φ_2 with the controllers in [20, 21] and in this paper; (c) the time response curves of auxiliary variables x_{12} and x_{22} with the controller in this paper

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