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FUZZY ADAPTIVE EVENT-TRIGGERED AND SELF-TRIGGERED CONTROL FOR STRICT-FEEDBACK NONLINEAR SYSTEMS WITH UNKNOWN CONTROL GAIN FUNCTIONS

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Abstract. *Fuzzy adaptive event-triggered and self-triggered control techniques are studied for uncertain nonlinear systems in this article. A new adaptive law is constructed by using the state of the system sampling time. Zeno behavior can be effectively removed by embedding a normal number in the event-triggered condition. Then, a self-triggered algorithm is constructed that uses the current state of the object to decide the next moment. Compared with existing algorithms, this algorithm is easier to express and calculate. Finally, it is proved that all states of the closed-loop system are semi-globally uniformly bounded. A simulation example shows the validity of the control scheme and the theory.*
Keywords: Event-triggered control (ETC), Self-triggered control (STC), Fuzzy logic systems, Backstepping design

1. **Introduction.** Recently, adaptive fuzzy backstepping control has received increasingly attention in the realm of control. For uncertain strict-feedback nonlinear systems [1-3], many excellent results have been obtained. The authors put forward several important adaptive neural network backstepping controllers in [1]. Although the adaptive state feedback controllers were designed in [2,3], the controlled object considered is limited to the strict-feedback nonlinear system with constant control gain. In order to conquer the above limitations, different methods have been proposed in [4-6]. By introducing some smoothing functions and bounded estimation methods in [4], the authors have successfully achieved the stability of the closed-loop system. The Nussbaum function method has been used to design the controller to overcome the issue of unknown control direction in [5]. [6] combined backstepping method with bounded control technology, and established a new Lyapunov function so that all variables of the closed-loop system are bounded.

Note that the above results are all traditional periodic control, ETC was developed due to the ability to save communication, computing, and power resources. A simple event-triggered scheduler has been studied in [7], which needs to wait T units after each transmission. In [8], the controller and event-triggered condition have been devised simultaneously to avoid ISS assumption. Specifically, [9] proposed a new switching ETC scheme, which enables the switching event-triggered controller to effectively offset nonlinearity, uncertainty and sampling errors. [10] presented a new model-free adaptive event-triggered control strategy for nonlinear discrete-time systems. In [11], event-triggered conditions were updated periodically. However, using ETC presents an obstacle: special hardware is usually required to continuously check trigger conditions. So, [12] proposed an STC mechanism. As far as we know, there is little work to research the self-triggered technique for nonlinear systems with unknown control gain function.

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Motivated by the corresponding research, this article investigates to study an event-triggered and a self-triggered techniques for nonlinear systems. The major contributions of this study are as below.

1) This is the first time to develop fuzzy adaptive self-triggered technology, which can effectively avoid continuous states monitoring. This process only needs the state information obtained at the moment when the current event is triggered, and there is no need to continuously monitor the state, which can save a large number of resources.

2) Unlike the existing results [13], it also needs to calculate the resources at discrete times. This paper effectively reduces the use of communication resources by embedding a constant in ETM.

3) A fuzzy adaptive law using only sampling time information is designed, which does not require real-time calculation and saves a lot of communication channel resources.

2. **Problem Statement and Preliminaries.**

2.1. **System descriptions and assumptions.**

Consider the following strict-feedback nonlinear systems:

$$\begin{aligned}
\dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)\, x_{i+1},\ 1 \le i \le n-1 \\
\dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)\, u \\
y &= x_1
\end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$, $\bar{x}_n = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the state variable, $y \in \mathbb{R}$ and $u \in \mathbb{R}$ are output and input of the nonlinear systems, respectively. $f(\cdot)$ and $g(\cdot)$ $(1 \le i \le n)$ are smooth unknown nonlinear functions.

Remark 2.1. *The system (1) is a common strict-feedback nonlinear system. In practice, many real-world systems can be modeled as the above nonlinear strict-feedback system, such as marine surface vehicle* [14] *and unmanned aerial vehicles system* [15].

Assumption 2.1. *There exist positive constants $g_{i,1}$, $g_{i,0}$ and $g_{i,d}$ which satisfy $g_{i,0} \le |g_i(\cdot)| \le g_{i,1}$ and $|\dot{g}_i(\cdot)| \le g_{i,d}$, $i = 1, 2, \ldots, n$.*

Lemma 2.1. *For any continuous function $f(x)$ defined over a compact set U and any given positive constant ε, there is an FLS $\theta^{*T}\varphi(x)$ such that*

$$\sup_{x \in U} \left| f(x) - \theta^{*T}\varphi(x) \right| \le \varepsilon \tag{2}$$

2.2. **Notion of event-triggered mechanism.**

Let $\{t_k\}_{k=1}^{\infty}$ with $t_{k+1} > t_k$ represent the event-triggered instants, and $x_i(t_k)$ be the state of the system at time instant t_k. Between successive event instant $[t_k, t_{k+1})$, the state vector is given as

$$\breve{x}_i(t) = x_i(t_k),\ t_k \le t \le t_{k+1} \tag{3}$$

An event error is defined as follows: $e_i(t) = x_i(t) - \breve{x}_i(t)$, $t_k \le t \le t_{k+1}$ which is determined of the moment of triggering. The controller has the following form $u(t) = u(t_k)$, $t \in [t_k, t_{k+1})$. For $i = 1, \ldots, n$ $\forall t \in [t_k, t_{k+1})$, the adaptive laws are described as

$$\dot{\hat{\theta}}_i(t) = \kappa_i z_i(t_k)\varphi_i(Z_i(t_k)) - \sigma_i\hat{\theta}_i(t) \tag{4}$$

where κ_i and σ_i are positive parameters.

3. **Event-Triggered Algorithm.**

3.1. **Event-triggered controllers design.**

This introduces the coordinate transformation:

$$\begin{aligned}
z_1 &= y - y_m \\
z_i &= x_i - \alpha_{i-1},\ i = 2, 3, \ldots, n
\end{aligned} \tag{5}$$

where α_{i-1} is the virtual control law.

Step 1: From (1) and (5), one can get

$$\begin{aligned}\dot{z}_1 &= g_1(x_1)\left(z_2+\alpha_1+\theta_1^{*T}\varphi(Z_1)+\varepsilon(Z_1)\right)\\ &= g_1(x_1)\left(z_2+\alpha_1+\hat{\theta}_1^T\varphi(Z_1)+\tilde{\theta}_1^T\varphi(Z_1)+\varepsilon(Z_1)\right)\end{aligned} \tag{6}$$

According to Lemma 2.1, let $h_1(Z_1) = g_1^{-1}(x_1)(f_1(x_1)-\dot{y}_m) = \theta_1^{*T}\varphi_1(Z_1)+\varepsilon_1(Z_1)$, $Z_1=[x_1,\dot{y}_m]^T$, $\varepsilon_1(Z_1)$ satisfied with $|\varepsilon_1(Z_1)|\le\varepsilon_1^*$, ε_1^* is a positive integer and $\tilde{\theta}_1=\theta_1^*-\hat{\theta}_1$ with $\hat{\theta}_1$ being the estimate of θ_1^*.

Choose the Lyapunov function candidate as $V_1=\frac{1}{2g_1(x_1)}z_1^2+\frac{1}{2\kappa_1}\tilde{\theta}_1^T\tilde{\theta}_1$, from (6), the time derivative of V_1 can be computed as

$$\dot{V}_1 = z_1z_2+z_1\left(\alpha_1+\hat{\theta}_1^T\varphi_1(Z_1)+\varepsilon_1^*-\frac{\dot{g}_1z_1}{2g_1^2(x_1)}\right)+\tilde{\theta}_1^Tz_1\varphi_1(Z_1)-\frac{1}{\kappa_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1 \tag{7}$$

To stabilize this subsystem, the event-triggered virtual control signal α_1 and the parameter adaptive law are designed as

$$\begin{aligned}\alpha_1 &= -c_1\breve{z}_1-\bar{c}_1\breve{z}_1-\hat{\theta}_1^T\varphi_1(Z_1)\\ \dot{\hat{\theta}}_1 &= \kappa_1z_1(t_k)\varphi_1(Z_1(t_k))-\sigma_1\hat{\theta}_1(t)\end{aligned} \tag{8}$$

where c_1 and $\bar{c}_1$ are positive parameters to be designed, and $\breve{z}_1=z_1(t_k)$, $t\in[t_k,t_{k+1})$ is an event-triggered variable. In addition, based on event error, we obtain $\breve{x}_1=x_1-e_1$; furthermore, one has $\breve{z}_1=z_1-e_1$. Substituting (8) into (7) gives

$$\begin{aligned}\dot{V}_1 \le{}& z_1z_2+z_1\left(-c_1\breve{z}_1-\bar{c}_1\breve{z}_1+\varepsilon_1^*-\frac{\dot{g}_1z_1}{2g_1^2(x_1)}\right)+z_1\tilde{\theta}_1^T\varphi_1(Z_1)-z_1(t_k)\tilde{\theta}_1^T\varphi_1(Z_1(t_k))\\ &+\frac{\sigma_1\tilde{\theta}_1^T\hat{\theta}_1}{\kappa_1}\end{aligned} \tag{9}$$

Using the Young's inequality, one gets

$$\begin{aligned}&z_1(t)\tilde{\theta}_1^T\varphi_1(Z_1(t))-z_1(t_k)\tilde{\theta}_1^T\varphi_1(Z_1(t_k))\\ &\le\tilde{\theta}_1^T(z_1(t)-z_1(t_k))\varphi_1(Z_1(t_k))+\tilde{\theta}_1^Tz_1(t)[\varphi_1(Z_1(t))-\varphi_1(Z_1(t_k))]\\ &\le\frac{\tilde{\theta}_1^T\tilde{\theta}_1}{2\kappa_1}+\kappa_1|z_1(t)-z_1(t_k)|^2+\kappa_1z_1^2\\ &z_1\varepsilon_1^*\le\frac{1}{2}z_1^2+\frac{1}{2}\varepsilon_1^{*2}\\ &\frac{\sigma_1\tilde{\theta}_1^T\hat{\theta}_1}{\kappa_1}\le\frac{\sigma_1\tilde{\theta}_1^T\left(\theta_1^*-\tilde{\theta}_1\right)}{\kappa_1}\le-\frac{\sigma_1}{2\kappa_1}\left\|\tilde{\theta}_1\right\|^2+\frac{\sigma_1}{2\kappa_1}\|\theta_1^*\|^2\end{aligned} \tag{10}$$

Further, substituting (10) into (9) gives

$$\dot{V}_1\le-\gamma_1z_1^2-\frac{(\sigma_1-1)}{2\kappa_1}\left\|\tilde{\theta}_1\right\|^2+(c_1+\bar{c}_1)z_1e_1+z_1z_2+\kappa_1|z_1(t)-z_1(t_k)|^2+D_1 \tag{11}$$

where $\bar{c}_1\ge\frac{1}{2}-\frac{\dot{g}_1}{2g_1^2(x_1)}$, $\gamma_1=c_1-\kappa_1$ and $D_1=\frac{\sigma_1}{2\kappa_1}\|\theta_1^*\|^2+\frac{1}{2}\varepsilon_1^{*2}$.

Step i $(i=2,\dots,n-1)$: From (1) and (5), one can obtain

$$\dot{z}_i=g_i(\bar{x}_i)\left(z_{i+1}+\alpha_i+\hat{\theta}_i^T\varphi_i(Z_i)+\tilde{\theta}_i^T\varphi_i(Z_i)+\varepsilon_i(Z_i)\right) \tag{12}$$

Let $h_i(Z_i) = g_i^{-1}(\bar{x}_i)(f_i(\bar{x}_i)-\dot{\alpha}_{i-1}+g_i(\bar{x}_i)z_{i-1}) = \theta_i^{*T}\varphi_i(Z_i)+\varepsilon_i(Z_i)$, $Z_i=\left[\bar{x}_i^T,\right.$ $\left.\partial\alpha_{i-1}/\partial x_1,\dots,\partial\alpha_{i-1}/\partial x_{i-1},\phi_{i-1}\right]^T$, $\varepsilon_i(Z_i)$ satisfied with $|\varepsilon_i(Z_i)|\le\varepsilon_i^*$. And $\tilde{\theta}_i=\theta_i^*-\hat{\theta}_i$ with $\hat{\theta}_i$ being the estimate of θ_i^*.

Choose the Lyapunov function candidate as $V_i = V_{i-1} + \frac{1}{2g_i(\bar{x}_i)} z_i^2 + \frac{1}{2\kappa_i}\tilde{\theta}_i^T\tilde{\theta}_i$, from (12), the time derivative of V_i can be computed as

$$\dot{V}_i \leq \dot{V}_{i-1} + z_i z_{i+1} + z_i\left(\alpha_i + \hat{\theta}_i^T\varphi_i(Z_i) + \varepsilon_i^* - \frac{\dot{g}_i z_i}{2g_i^2(x_i)}\right) + \tilde{\theta}_i^T z_i\varphi_i(Z_i) - \frac{\tilde{\theta}_i^T\dot{\hat{\theta}}_i}{\kappa_i} \tag{13}$$

Choose the virtual control signal and the adaptive law as

$$\begin{aligned} \alpha_i &= -c_i\breve{z}_i - \bar{c}_i\breve{z}_i - \hat{\theta}_i^T\varphi_i(Z_i) \\ \dot{\hat{\theta}}_i &= \kappa_i z_i(t_k)\varphi_i(Z_i(t_k)) - \sigma_i\hat{\theta}_i(t) \end{aligned} \tag{14}$$

where c_i and $\bar{c}_i$ are positive parameters to be designed, and $\breve{z}_i = z_i(t_k)$, $t \in [t_k, t_{k+1})$ is an event-triggered variable. We can obtain $\breve{x}_i = x_i - e_i$, which indicates $\breve{z}_i = z_i - e_i$.

Then, one can obtain

$$\begin{aligned} \dot{V}_i \leq{}& \dot{V}_{i-1} + z_i z_{i+1} + z_i\left(-c_i\breve{z}_i - \bar{c}_i\breve{z}_i + \varepsilon_i^* - \frac{\dot{g}_i z_i}{2g_i^2(x_i)}\right) + \tilde{\theta}_i^T z_i\varphi_i(Z_i) \\ &- \tilde{\theta}_i^T z_i(t_k)\varphi_i(Z_i(t_k)) + \frac{\tilde{\theta}_i^T\dot{\hat{\theta}}_i}{\kappa_i} \end{aligned} \tag{15}$$

By using the Young's inequality, the algorithm is the same as step 1. Further, one has

$$\begin{aligned} \dot{V}_i \leq{}& -\sum_{k=1}^{i}\gamma_k z_k^2 - \sum_{k=1}^{i}\frac{(\sigma_k - 1)}{2}\left\|\tilde{\theta}_k\right\|^2 + \sum_{k=1}^{i}(c_k + \bar{c}_k)z_k e_k \\ &+ \sum_{k=1}^{i}\kappa_k\left|z_i(t) - z_i(t_k)\right|^2 + z_i z_{i+1} + D_i \end{aligned} \tag{16}$$

where $\bar{c}_i \geq \frac{1}{2} - \frac{\dot{g}_i}{2g_i^2(x_i)}$, $\gamma_k = c_k - \kappa_k$ and $D_i = D_{i-1} + \frac{\sigma_i}{2\kappa_i}\left\|\theta_i^*\right\|^2 + \frac{1}{2}\varepsilon_i^{*2}$.

Step n: With the same design procedure in step i, choose the control signal and the adaptive law as

$$\begin{aligned} u &= -c_n\breve{z}_n - \bar{c}_n\breve{z}_n - \hat{\theta}_n^T\varphi_n(Z_n) \\ \dot{\hat{\theta}}_n &= \kappa_n z_n(t_k)\varphi_n(Z_n(t_k)) - \sigma_n\hat{\theta}_n(t) \end{aligned} \tag{17}$$

where c_n and $\bar{c}_n$ are positive parameters to be designed, and $\breve{z}_n = z_n(t_k)$, $t \in [t_k, t_{k+1})$ is an event-triggered variable. By (17), one can obtain

$$\begin{aligned} \dot{V}_n ={}& \dot{V}_{n-1} + z_n\left(-c_n\breve{z}_n - \bar{c}_n\breve{z}_n + \varepsilon_n^* - \frac{\dot{g}_n z_n}{2g_n^2(\bar{x}_n)}\right) + \tilde{\theta}_n^T\varphi_n(Z_n)z_n \\ &- \tilde{\theta}_n^T z_n(t_k)\varphi_n(Z_n(t_k)) + \frac{\sigma_n\tilde{\theta}_n^T\hat{\theta}_n}{\kappa_n} \end{aligned} \tag{18}$$

By utilizing the Young's inequality, the algorithm is the same as step 1. Further, one has

$$\begin{aligned} \dot{V}_n \leq{}& -\sum_{k=1}^{n}\gamma_k z_k^2 - \sum_{k=1}^{n}\frac{(\sigma_k - 1)}{2}\left\|\tilde{\theta}_k\right\|^2 + \sum_{k=1}^{n}(c_k + \bar{c}_k)z_k e_k \\ &+ \sum_{k=1}^{n}\kappa_k\left|z_k(t) - z_k(t_k)\right|^2 + D_n \end{aligned} \tag{19}$$

where $\bar{c}_n \geq \frac{1}{2} - \frac{\dot{g}_n}{2g_n^2(x_n)}$, $\gamma_n = c_n - \kappa_n$ and $D_n = \sum_{k=1}^{n}\frac{\sigma_k}{2\kappa_k}\left\|\theta_k^*\right\|^2 + \sum_{k=1}^{n}\frac{1}{2}\varepsilon_k^{*2}$.

Remark 3.1. *For $\forall t \in [t_k, t_{k+1})$, since the controller contains system state and adaptive law, and they are sampled at instant t_k, the adaptive law and state are triggered at the same time. Compared with [11], we propose a method to calculate the adaptive law only at t_k time, which greatly saves network resources.*

Now, the event-triggered mechanism is given as

$$t_{k+1} = t_k + \max\{\tau_k, b_i\} \tag{20}$$

where b_i is a strictly positive real number, given in the following Theorem 3.1, and τ_k is described as

$$\tau_k = \inf_{t>t_k}\{t - t_k|\,|e_i(t)| > \phi_i\xi_i\,|z_i(t)|\} \tag{21}$$

with $\|\cdot\|$ being the Euclidean norm and $0 < \phi_i < 1$, $\xi_i > 0$.

3.2. **Stability analysis.**

Theorem 3.1. *Considering nonlinear system given by (1), event-triggered controllers and adaptive laws (8), (14), (17), there exists* b_i*, which is strictly positive satisfying*

$$b_i \le \frac{\frac{\sqrt{\gamma_i}}{\sqrt{c_i+\bar{c}_i}}}{\bar{\zeta}\left(1+\frac{\sqrt{\gamma_i}}{\sqrt{c_i+\bar{c}_i}}\right)} \tag{22}$$

Then, all the signals of system (1) are bounded and the tracking error will exponentially converge to a residual. Moreover, there is no Zeno behavior.

Proof: To find the lower time bound b_i, the time derivative of $\frac{\|e_i(t)\|}{\|z_i(t)\|}$ is

$$\dot{z} = \dot{z}_i = -c_i(z_i - e_i) - \bar{c}_i(z_i - e_i) + \frac{1}{2}z_i - \frac{\dot{g}_i z_i}{2g_i^2(\bar{x}_i)} \le (c_i + \bar{c}_i)\|z_i + e_i\| \tag{23}$$

Then one can get

$$\begin{aligned} \frac{d}{dt}\frac{\|e_i\|}{\|z\|} &\le \frac{e^T\dot{e}}{\|e\|\|z\|} - \frac{\|e\|z^T\dot{z}}{\|z\|^3} \le \frac{\|\dot{z}\|}{\|z\|} + \frac{\|e\|}{\|z\|}\cdot\frac{\|\dot{z}\|}{\|z\|} \\ &\le \left(1+\frac{\|e\|}{\|z\|}\right)(c_i+\bar{c}_i)\frac{\|z_i+e_i\|}{\|z\|} = \bar{\zeta}\left(1+\frac{\|e\|}{\|z\|}\right)^2 \end{aligned} \tag{24}$$

where $\bar{\zeta} = c_i + \bar{c}_i$.

It is noted that $\frac{\|e_i\|}{\|z\|}$ is always upper bounded by $\frac{\|e\|}{\|z\|}$ and both of them are nonnegative. Then we conclude that $\frac{\|e_i\|}{\|z\|}$ satisfies the bound $\frac{\|e\|}{\|z\|} < y(t, y_0)$ where $y(t, y_0)$ is the solution of $\dot{y}(t) = \bar{\zeta}(1 + y(t))^2$, $y_0 = 0$. Then the evolution time of $\frac{\|e_i\|}{\|z\|}$ from 0 to $\sqrt{\xi_i}$ is lower bounded by

$$B_i = \frac{\sqrt{\xi_i}}{\bar{\zeta}\left(1+\sqrt{\xi_i}\right)} = \frac{\frac{\sqrt{\gamma_i}}{\sqrt{c_i+\bar{c}_i}}}{\bar{\zeta}\left(1+\frac{\sqrt{\gamma_i}}{\sqrt{c_i+\bar{c}_i}}\right)} \tag{25}$$

Further, we consider the stability of the system

$$\dot{V}_n \le -\sum_{k=1}^{n}\gamma_k z_k^2 - \sum_{k=1}^{n}\frac{(\sigma_k-1)}{2}\left\|\tilde{\theta}_k\right\|^2 + \sum_{k=1}^{n}(c_k+\bar{c}_k)z_k e_k + \sum_{k=1}^{n}\kappa_k e_k^2 + D_n \tag{26}$$

According to (26), where ξ_i is chosen as $\xi_i = \frac{\gamma_i}{c_i+\bar{c}_i}$, then, one has

$$\dot{V}_n \le -\sum_{k=1}^{n}\left(1-\phi-\kappa_k\left(\frac{\phi\gamma_k}{c_k+\bar{c}_k}\right)^2\right)z_k^2 + \sum_{k=1}^{n}\frac{(\sigma_k-1)}{2}\left\|\tilde{\theta}_k\right\|^2 + D_n \tag{27}$$

Let $C = \min\left\{2g_{i,0}\left(1-\phi-\kappa_k\left(\frac{\phi\gamma_k}{c_k+\bar{c}_k}\right)^2\right), (\sigma_i-1)\Big/2\right\}$, and we get

$$\dot{V} \le -CV + D \tag{28}$$

where $D = D_n = \sum_{k=1}^{n} \frac{\sigma_k}{2\kappa_k}\|\theta_k^*\|^2 + \sum_{k=1}^{n} \frac{1}{2}\varepsilon_k^{*2}$. Then, (26) becomes $V_n(t) \le V(t)\big(V(0) - \frac{D}{C}\big)\, e^{-Ct} + \frac{D}{C}$. Further, $\forall t \in [t_k, t_{k+1})$, when $t \to \infty$, one has

$$\lim_{t\to\infty} V(t) \le \frac{2D}{C} \tag{29}$$

Furthermore, we obtain $\lim_{t\to\infty}|z_i| \le 2\sqrt{(D/C)}$ and $\lim_{t\to\infty}\left|\tilde{\theta}_i\right| \le 2\sqrt{(\kappa_i D/C)}$.

From (29), we obtain $e_i^2(t) \le V(t) \le V(0)e^{-Ct}$. Thus, all states of system (1) are bounded.

Remark 3.2. *From the event-triggered condition (19), the error is corrected with the states. ϕ and ξ are regulated by γ, c and $\bar{c}$, to guarantee the closed-loop stability such that the parameters are chosen $c = 60$, $\bar{c} = 2$, $\phi = 0.9$, $\kappa = 0.1$. Through calculation, we can get $\kappa_k\left(\frac{\phi\gamma_k}{c_k+\bar{c}_k}\right)^2$ is 0.076, and $1 - \phi - \kappa_k\left(\frac{\phi\gamma_k}{c_k+\bar{c}_k}\right)^2$ is always bigger than zero. Therefore, it does not affect the stability of the system.*

4. **Self-Triggered Algorithm.**

The self-triggered policy is given in Theorem 4.1.

Theorem 4.1. *In order to construct self-triggered algorithm of the closed-loop system (1), the inter-execution interval function $\tau_k(x(t_k)) = t_{k+1} - t_k$ is considered as*

$$\tau_k(x(t_k)) = t_{k+1} - t_k = \frac{\phi_i\xi_i}{(1+\phi_i\xi_i)M_1}|z_i(t_k)| \tag{30}$$

where $M_1 \ge \kappa_i\sqrt{V(0)}$.

Proof: Using (3), we have $|\dot{e}_i(t)| = |\dot{z}_i(t)| \le M_1 e_i(t)$. Further, we can get $|\dot{e}_i(t)| \le M_1$ with M_1 shown in (30), one has $|e_i(t)| \le |e(t)| \le M_1 t$. Because the continuity of $|e_i(t)|$ and $|z_i(t)|$, for any $T \in (t_k, t_{k+1})$, $|e_i(t)|$ can only decrease to a level which is bigger than $\phi_i\xi_i|z_i(t)|$ from time t_k to T. To get a valid T, the value of $V(0)$ is needed to be memorized by state x_1 during the evolution. Since $e_i(t) = x_i(t_k) - x_i(t)$, a sufficient condition to guarantee $|e_i(t)| > \phi_i\xi_i|z_i(t)|$ is $|e_i(t)| > \frac{\phi_i\xi_i}{1+\phi_i\xi_i}|z_i(t_k)|$. Let T' be the accurate time consumed for $|e_i(t)|$ to increase from 0 to $\frac{\phi_i\xi_i}{1+\phi_i\xi_i}|z_i(t_k)|$. From (30), we denote $T_0 = \frac{\phi_i\xi_i}{(1+\phi_i\xi_i)M_1}|z_i(t_k)|$. Then, $\tilde{T} = t_k + T_0 < T' < t_{k+1}$ will be a feasible choice for $T = t_{k+1}$ to ensure $|e_i(t)| > \phi_i\xi_i|z_i(t)|$ holds. It means that the inter-event time can be chosen as $\tau_k = \frac{\phi_i\xi_i}{(1+\phi_i\xi_i)M_1}|z_i(t_k)|$. This completes the proof.

5. **Simulation Results.**

Example 5.1. *Consider the following two-order nonlinear system:*

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(\bar{x}_2) + g_2(\bar{x}_2)u \\
y &= x_1
\end{aligned} \tag{31}$$

The design parameters are chosen as $c_1 = 60$, $\bar{c}_1 = 2$, $c_2 = 60$, $\bar{c}_2 = 3$, $\sigma_1 = \sigma_2 = 0.2$, $\phi_1 = 0.9$, $\phi_2 = 0.9$, $\xi_1 = 0.5$, $\xi_2 = 0.4$. The initial values of the variables and adaptive parameters are chosen as $x_1(0) = 0.2$, $x_2(0) = 0$, $x_3(0) = 0.4$, $x_4(0) = 1$, $\theta_1(0) = \theta_2(0) = [0, 0, 0, 0, 0]^T$, and the other initial values are chosen zeros.

Figure 1 shows the event times which is the event-triggered algorithm compared with the self-triggered algorithm. It is obvious that the number of self-triggered methods is much less than that of event-triggered, which proves the effectiveness of the self-triggered algorithm.

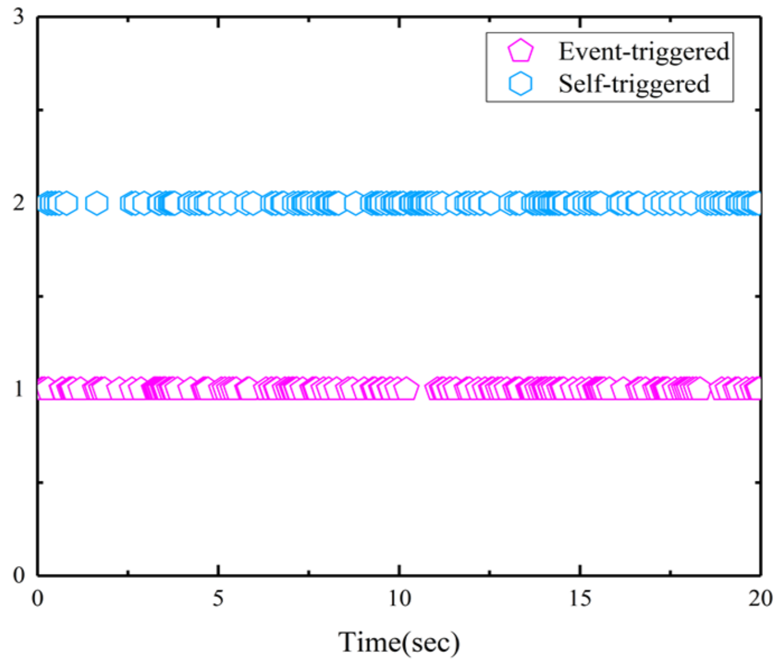

FIGURE 1. Triggering event

6. **Conclusions.** Adaptive fuzzy event-triggered and self-triggered schemes for uncertain strict-feedback nonlinear systems are investigated in this article. A new adaptive law solves the issue of event-triggered in fuzzy adaptive control. It is worth mentioning that the self-triggered technique further reduces the waste of event monitoring, in which the next triggered moment of communication can be decided by the state of the current moment. In the future, it will be a meaningful research direction to apply the proposed self-triggered method to output-feedback control methods and interconnected large-scale systems.

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