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PREDEFINED TIME TRAJECTORY TRACKING ADAPTIVE CONTROL FOR UNDERACTUATED AUTONOMOUS UNDERWATER VEHICLES

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Abstract. *This paper concentrates on the issue of adaptive predefined-time trajectory tracking control for underactuated autonomous underwater vehicles (AUVs). With the help of time-varying asymmetric barrier function, the good trajectory tracking accuracy of AUVs can be achieved. Based on predefined-time stable and adaptive backstepping control, a novel predefined-time trajectory tracking adaptive constraint control approach is developed. The proposed control approach can ensure the AUV's trajectory tracking error converges to a preset error constrained region in predefined time, and also guarantees the other closed-loop signals are bounded in predefined time.*
Keywords: Underactuated AUVs, Adaptive trajectory tracking control, Backstepping control, Predefined-time control, Time-varying asymmetric barrier function

1. **Introduction.** With the rapid development of marine techniques, underactuated AUVs or autonomous vehicles, as a more efficient tool for exploring ocean boundaries, have attracted considerable attention for scholars. Meanwhile, many useful works have been published, for example, [1-3]. The authors in [1] investigate a leader-follower formation control for multi-underactuated AUVs, and the dynamic model and kinematic of AUVs are given. Inspired by [1], [2] develops a robust adaptive trajectory tracking control method for AUVs. By adopting prescribed performance technique, the trajectory tracking accuracy of AUVs is ensured. Then, the authors in [3] study the trajectory following control method for autonomous vehicles by designing a feedforward controller. Note that the above developed control approaches do not ensure the settling time of AUVs usually tends to infinite, which will be detrimental to the development of marine exploring industry. Thus, the high trajectory tracking accuracy and the faster settling time need to be required.

To overcome this drawback, the fixed-time stable theory is developed for nonlinear systems in [4]. The settling time in [4] does not depend on the initial values, but only on the design parameters. Thus, inspired by [4], the author in [5] investigates the fixed-time adaptive output constraint control issue for nonlinear systems. The authors in [6] develop an adaptive fuzzy fixed-time control approach for nonlinear systems, and the concept of practical fixed-time stable is developed. In addition, [7,8] study adaptive fixed-time output feedback control issues for nonlinear systems. The authors in [9] study a robust adaptive practical fixed-time leader-follower formation control for AUVs. Note that [8,9]

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also develop the non-singular fixed-time control methods, but the settling time depends on the small design parameter.

Obviously, the fixed-time control schemes in [6-9] are all depending on a small design parameter. Thus, a novel predefined-time adaptive control approach is developed for robotic in [10], and the settling-time is given in advance. Then, the authors in [11] develop a singularity-free adaptive predefined-time control approach for rigid spacecrafts. However, it should be noted that there are not available works about the predefined-time adaptive constraint tracking control for AUVs.

Inspired by the above discussions, this paper concentrates on the predefined-time-based adaptive trajectory tracking constraint control issue for underactuated AUVs. Compared with the existing works, the main contributions of this paper can be highlighted as follows.

1) With the help of predefined-time stable theory and hyperbolic tangent function, a predefined-time adaptive trajectory tracking control approach is developed for AUVs. The problem that the settling time depends on the small design parameter and singular problem are solved in [8,9].

2) An output feedback control approach is developed for AUVs by using time-varying barrier function. The developed constraint control approach can ensure the tracking error converges to a preset time-varying error constraint region, and then the trajectory tracking accuracy can be ensured.

2. **Problem Statement and Preliminaries.**

2.1. **Model of AUVs.**

Based on the body and earth fixed coordinates, inspired by [1,2], AUV usually can be modeled as follows:

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{R}(\psi)\upsilon \\ \boldsymbol{\tau} + \mathbf{d}(t) &= \mathbf{M}\dot{\upsilon} + \mathbf{C}(\upsilon)\upsilon + \mathbf{D}(\upsilon)\upsilon \end{aligned} \tag{1}$$

where $\upsilon = [u, v, r]^{\mathrm{T}}$ represents the AUV's velocity vector with yaw rate r, sway velocity v and surge velocity u; $\boldsymbol{\eta} = [x, y, \psi]^{\mathrm{T}}$ represents the AUV's position vector with yaw angle $\psi \in [0, 2\pi)$ and position (x, y); $\mathbf{d}(t) = [d_1(t), d_2(t), d_3(t)]^{\mathrm{T}}$ represents external disturbance vector; $\boldsymbol{\tau} = [\tau_u, 0, \tau_r]^{\mathrm{T}}$ represents the control input vector with yaw moment, sway force and surge force. The damping matrix $\mathbf{D}(\upsilon)$, the matrix of Coriolis and centripetal terms $\mathbf{C}(\upsilon)$, positive definite inertia matrix $\mathbf{M}$ and rotation matrix $\mathbf{R}(\psi)$ are described as

$$\mathbf{D}(\upsilon) = \begin{bmatrix} d_{1,1}(u) & 0 & 0 \\ 0 & d_{2,2}(v) & 0 \\ 0 & 0 & d_{3,3}(r) \end{bmatrix}, \quad \mathbf{C}(\upsilon) = \begin{bmatrix} 0 & 0 & -m_{2,2}v \\ 0 & 0 & m_{1,1}u \\ m_{2,2}v & -m_{1,1}u & 0 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & 0 & 0 \\ 0 & m_{2,2} & 0 \\ 0 & 0 & m_{3,3} \end{bmatrix}, \quad \mathbf{R}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $d_{1,1}(u) = -X_u - X_{u|u|}|u|$, $d_{2,2}(v) = -Y_v - Y_{v|v|}|v|$ and $d_{3,3}(r) = -N_r - N_{r|r|}|r|$ with hydrodynamic derivatives $X_u, X_{u|u|}$, $Y_v, Y_{v|v|}$, $N_r, N_{r|r|}$; $m_{1,1} = m - X_{\dot{u}}$, $m_{2,2} = m - Y_{\dot{v}}$, $m_{3,3} = I_z - N_{\dot{r}}$ with AUV's mass m, added masses $X_{\dot{u}}$, $Y_{\dot{v}}$ and $N_{\dot{r}}$, and moment of inertia in yaw I_z.

Assumption 2.1. [1] *The desired trajectory* $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^{\mathrm{T}}$ *is bounded,* $\dot{\boldsymbol{\eta}}_d$ *and* $\ddot{\boldsymbol{\eta}}_d$ *are also bounded.*

Assumption 2.2. [2] *There exists unknown constant vector* $d^* = \left[d_1^*, d_2^*, d_3^*\right]^{\mathrm{T}}$*, and the external disturbance vector* $\mathbf{d}(t) = \left[d_1(t), d_2(t), d_3(t)\right]^{\mathrm{T}}$ *satisfies* $\|\mathbf{d}(t)\| \leq \|d^*(t)\|$.

Control Objective: This paper will design an adaptive predefined-time constraint tracking control law for AUVs (1) such that

1) AUV can track the desired trajectory $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^{\mathrm{T}}$ in predefined time;

2) All closed-loop signals are bounded;

3) Tracking error e_j $(j = x, y, \psi)$ are not beyond the constraint regions $\overline{\mathcal{H}}_j(t)$ and $\underline{\mathcal{H}}_j(t)$. e_j, $\underline{\mathcal{H}}_j(t)$ and $\overline{\mathcal{H}}_j(t)$ will be defined in Section 3.1.

2.2. **Preliminaries.** The following useful knowledge is introduced to achieve the control objective of this paper.

Definition 2.1. [10, 11] *For nonlinear system* $\dot{\chi}(t) = f(\chi, t)$ *with* $f(0) = 0$*, the equilibrium point* $\chi(0) = \chi_0$ *is said to be the practical predefined-time stable (PTS) if the state trajectory satisfies* $\|\chi(\chi_0, t)\| \le \delta$ *for* $\forall t \ge \mathrm{T}_{\max}$ *with predefined time* $\mathrm{T}_{\max}$ *and constant* $\delta > 0$*. Thus, for any constants* $\beta \in (0, 1)$*,* $\epsilon > 0$*,* $\bar{\epsilon} > 0$ *and* $D > 0$*, there exists a continuous differentiable function* $V(\chi)$*, we have*

$$\dot{V}(\chi) \le -\frac{\bar{\epsilon}\pi}{\beta \mathrm{T}_{\max}} V^{1+\frac{\beta}{2}} - \frac{\epsilon\pi}{\beta \mathrm{T}_{\max}} V^{1-\frac{\beta}{2}} + D \tag{2}$$

thus, the nonlinear system $\dot{\chi}(t) = f(\chi, t)$ *is practical predefined-time stable (PPTS).*

Lemma 2.1. [7] *Assume that* $\left|\tilde{\vartheta}\right| \le \delta^*$ *with constant* $\delta^* > 0$*; thus for constants* $\beta_1 \in \left(\frac{1}{2}, 1\right)$ *and* $\beta_2 > 1$*, the following inequality holds*

$$-\tilde{\vartheta}^{\mathrm{T}}\tilde{\vartheta} \le \left(\frac{1}{2}\tilde{\vartheta}^{\mathrm{T}}\tilde{\vartheta}\right)^{\beta_1} - \left(\frac{1}{2}\tilde{\vartheta}^{\mathrm{T}}\tilde{\vartheta}\right)^{\beta_2} + \overline{\Gamma} \tag{3}$$

where $\overline{\Gamma} = (1 - \beta_1)\beta_1^{\frac{\beta_1}{1-\beta_1}} + \left(\frac{\delta^*}{2}\right)^{\beta_2}$.

Lemma 2.2. [6] *For* $\chi_i \in R$*, there exist constants* $p \in (0, 1]$ *and* $q > 1$*, and we have*

$$\sum_{i=1}^{m} |\chi_i|^q \ge m^{1-q} \left(\sum_{i=1}^{m} |\chi_i|\right)^q, \quad \sum_{i=1}^{m} |\chi_i|^p \ge \left(\sum_{i=1}^{m} |\chi_i|\right)^p \tag{4}$$

3. **Main Results.** In this section, the adaptive path following control law will be designed, and the stability analysis will be given.

3.1. **Adaptive predefined time path following controller design.** In this subsection, an adaptive predefined time tracking controller will be designed by using the universal asymmetric barrier function and novel predefined-time technique.

First, define the tracking error $\mathbf{e} = [e_x, e_y, e_\psi]^{\mathrm{T}}$ as

$$\mathbf{e} = \boldsymbol{\eta} - \boldsymbol{\eta}_\alpha \tag{5}$$

where $\boldsymbol{\eta}_\alpha = [x_d, y_d, \psi_\alpha]^{\mathrm{T}}$, ψ_α is called to be approach angle, which is used to solve the AUV's underactuation issue, and defined as

$$\psi_\alpha = \mathrm{atan2}(e_y, e_x) \tanh\left(\frac{e_x^2 + e_y^2}{\delta}\right) + \psi_d \left(1 - \tanh\left(\frac{e_x^2 + e_y^2}{\delta}\right)\right) \tag{6}$$

Obviously, $\psi_\alpha = \psi_d$ when $e_x = e_y = 0$.

To achieve the predefined-time constraint control objective, according to [5] and [12], define the following universal asymmetric barrier function as

$$z_{1,j} = \mathrm{T}\left(e_j, \overline{\mathcal{H}}_j, \underline{\mathcal{H}}_j\right) = \frac{\overline{\mathcal{H}}_j \underline{\mathcal{H}}_j e_j}{\left(\overline{\mathcal{H}}_j - e_j\right)\left(\underline{\mathcal{H}}_j + e_j\right)}, \quad -\underline{\mathcal{H}}_j(0) < e_j(0) < \overline{\mathcal{H}}_j(0) \tag{7}$$

where $\overline{\mathcal{H}}_j(t) > 0$ and $\underline{\mathcal{H}}_j(t) > 0$ are time-varying continuous functions, and $\overline{\mathcal{H}}_j(t) \ne \underline{\mathcal{H}}_j(t)$ $(j = x, y, \psi)$. Obviously, $z_{1,j} = 0$ if and only if $e_j = 0$. Then, when $e_j \to \overline{\mathcal{H}}_j$ or $e_j \to -\underline{\mathcal{H}}_j$, we have $z_{1,j} \to +\infty$ or $z_{1,j} \to -\infty$.

Hereafter, the adaptive backstepping-based predefined-time constraint controller design will be given.

Step 1: From (5) and (7), $\dot{z}_{1,j}$ is

$$\dot{z}_{1,j} = \mathcal{P}_j \dot{e}_j + \mathcal{Q}_j \tag{8}$$

where $\mathcal{P}_j = \frac{\overline{\mathcal{H}}_j \underline{\mathcal{H}}_j \left(e_j^2 + \overline{\mathcal{H}}_j \underline{\mathcal{H}}_j\right)}{\left(\overline{\mathcal{H}}_j - e_j\right)\left(\underline{\mathcal{H}}_j + e_j\right)}$, $\mathcal{Q}_j = -\frac{\underline{\mathcal{H}}_j e_j^2}{\left(\overline{\mathcal{H}}_j - e_j\right)^2\left(\underline{\mathcal{H}}_j + e_j\right)} \dot{\overline{\mathcal{H}}}_j + \frac{\overline{\mathcal{H}}_j e_j^2}{\left(\overline{\mathcal{H}}_j - e_j\right)\left(\underline{\mathcal{H}}_j + e_j\right)^2} \dot{\underline{\mathcal{H}}}_j$.

From (1) and (5), we have $\dot{e}_j = \mathbf{R}(\psi)\upsilon - \dot{\boldsymbol{\eta}}_\alpha$, define $\mathbf{z}_2 = \upsilon - \boldsymbol{\alpha}$, and thus $\dot{\mathbf{z}}_1$ is

$$\dot{\mathbf{z}}_1 = \mathbf{P}\left[\mathbf{R}(\psi)(\mathbf{z}_2 + \boldsymbol{\alpha}) - \dot{\boldsymbol{\eta}}_\alpha\right] + \mathbf{Q} \tag{9}$$

where $\mathbf{P} = \operatorname{diag}\left(\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_\psi\right)$, $\mathbf{Q} = \left[\mathcal{Q}_x, \mathcal{Q}_y, \mathcal{Q}_\psi\right]^{\mathrm{T}}$.

Choose the Lyapunov function as

$$V_1 = \frac{1}{2}\mathbf{z}_1^{\mathrm{T}}\mathbf{z}_1 \tag{10}$$

From (9) and (10), $\dot{V}_1$ is

$$\dot{V}_1 = \mathbf{z}_1^{\mathrm{T}}\dot{\mathbf{z}}_1 = \mathbf{z}_1^{\mathrm{T}}\mathbf{P}\left[\mathbf{R}(\psi)(\mathbf{z}_2 + \boldsymbol{\alpha}) - \dot{\boldsymbol{\eta}}_\alpha\right] + \mathbf{z}_1^{\mathrm{T}}\mathbf{Q} \tag{11}$$

With the help of Young's inequality, the following inequality holds

$$\mathbf{z}_1^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 \le \mathbf{z}_1^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_1 + \mathbf{z}_2^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 \tag{12}$$

Design the virtual control law $\boldsymbol{\alpha}$ as

$$\begin{aligned}\boldsymbol{\alpha} = \mathbf{R}^{-1}(\psi)\Bigg\{\mathbf{P}^{-1}\Bigg[&-\frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{1+\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\mathbf{z}_1^{1-\beta} \\ &\times\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\tanh\left(\frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\frac{\mathbf{z}_1^{2-\beta}}{\varsigma_1}\right) - \mathbf{Q}\Bigg] + \dot{\boldsymbol{\eta}}_\alpha\Bigg\} - \mathbf{R}(\psi)\mathbf{z}_1\end{aligned} \tag{13}$$

where $\varsigma_1 > 0$ is a constant.

Invoking (11)-(13), $\dot{V}_1$ can be further written as

$$\begin{aligned}\dot{V}_1 \le\ & \mathbf{z}_2^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 - \frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{2+\beta} \\ & - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta}\tanh\left(\frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\frac{\mathbf{z}_1^{2-\beta}}{\varsigma_1}\right)\end{aligned} \tag{14}$$

Inspired by [9], the inequality $|\chi| - \chi\tanh\left(\frac{\chi}{\varsigma}\right) \le \kappa\varsigma$ is used to handle term $\tanh(\cdot)$; thus, the following inequality holds

$$\frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta}\tanh\left(\frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\frac{\mathbf{z}_1^{2-\beta}}{\varsigma_1}\right) \le \kappa\varsigma_1 \tag{15}$$

where $\kappa = 0.2785$. Thus, $\dot{V}_1$ can be finally written as

$$\dot{V}_1 \le \mathbf{z}_2^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 - \frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{2+\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta} + \kappa\varsigma_1 \tag{16}$$

Step 2: From $\mathbf{z}_2 = \upsilon - \boldsymbol{\alpha}$, $\dot{\mathbf{z}}_2$ is

$$\dot{\mathbf{z}}_2 = \mathbf{M}^{-1}\left[\boldsymbol{\tau} + \mathbf{d}(t) - \mathbf{C}(\upsilon)\upsilon - \mathbf{D}(\upsilon)\upsilon - \mathbf{M}\dot{\boldsymbol{\alpha}}\right] \tag{17}$$

Define $\mathbf{F}\left(\upsilon^{\mathrm{T}}, \dot{\boldsymbol{\alpha}}\right)$ as

$$\mathbf{F}(\mathbf{Z}) = -\mathbf{M}^{-1}\left[\mathbf{C}(\upsilon)\upsilon + \mathbf{D}(\upsilon)\upsilon + \mathbf{M}\dot{\boldsymbol{\alpha}}\right] \tag{18}$$

where $\mathbf{Z} = \left[\upsilon^{\mathrm{T}}, \dot{\boldsymbol{\alpha}}\right]^{\mathrm{T}}$.

In the practical environment, there exist the uncertain parameters in AUVs; thus, with the help of universal approximation property of FLS in [6], an FLS $\hat{\mathbf{F}}\left(\mathbf{Z}\middle|\hat{\boldsymbol{\theta}}\right) = \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z})$ is adopted to identify $\mathbf{F}(\mathbf{Z})$, and assume that

$$\mathbf{F}(\mathbf{Z}) = \boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z}) + \boldsymbol{\varepsilon}(\mathbf{Z}) \tag{19}$$

where $\boldsymbol{\varphi}(\mathbf{Z}) = [\varphi_1(\mathbf{Z}), \varphi_2(\mathbf{Z}), \varphi_3(\mathbf{Z})]^{\mathrm{T}}$ is the fuzzy basis function vector, $\boldsymbol{\varepsilon}(\mathbf{Z}) = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^{\mathrm{T}}$ is the identify error vector, and $\boldsymbol{\theta}^*$ is the ideal weight vector, which is defined as

$$\boldsymbol{\theta}^{*\mathrm{T}} = \begin{bmatrix} \theta_1^{*\mathrm{T}} & 0 & 0 \\ 0 & \theta_2^{*\mathrm{T}} & 0 \\ 0 & 0 & \theta_3^{*\mathrm{T}} \end{bmatrix}$$

Thus, $\dot{\mathbf{z}}_2$ can be further rewritten as

$$\dot{\mathbf{z}}_2 = \mathbf{M}^{-1}\left[\boldsymbol{\tau} + \mathbf{d}(t) + \boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z}) + \boldsymbol{\varepsilon}(\mathbf{Z})\right] \tag{20}$$

Choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\mathbf{z}_2^{\mathrm{T}}\mathbf{M}\mathbf{z}_2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} \tag{21}$$

where $\boldsymbol{\Gamma} \in R^{3\times 3}$ is a positive-definite gain matrix. $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta}^* - \hat{\boldsymbol{\theta}}$ is the estimation error, and $\hat{\boldsymbol{\theta}}$ is the estimation of $\boldsymbol{\theta}^*$.

Invoking (20) and (21), $\dot{V}_2$ is

$$\dot{V}_2 = \dot{V}_1 + \mathbf{z}_2^{\mathrm{T}}\left[\boldsymbol{\tau} + \mathbf{d}(t) + \boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z}) + \boldsymbol{\varepsilon}(\mathbf{Z})\right] - \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} \tag{22}$$

Based on Assumption 2.2, define $\boldsymbol{\omega}_{1,0}^* = \boldsymbol{\varepsilon}(\mathbf{Z}) + \mathbf{d}(t)$, there is an unknown constant $\omega_{1,0} > 0$, $\boldsymbol{\omega}_{1,0}^*$ satisfies $\left\|\boldsymbol{\omega}_{1,0}^*\right\| \le \omega_{1,0}$. Then, similar to (15), we have

$$\mathbf{z}_2^{\mathrm{T}}\boldsymbol{\omega}_{1,0}^* - \omega_{1,0}\mathbf{z}_2\tanh\left(\frac{\omega_{1,0}\mathbf{z}_2}{\varsigma_2}\right) \le \kappa\varsigma_2 \tag{23}$$

Invoking (21)-(23), $\dot{V}_2$ can be further written as

$$\begin{aligned}\dot{V}_2 \le & -\frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{2+\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta} + \mathbf{z}_2^{\mathrm{T}}\mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 \\ & + \mathbf{z}_2^{\mathrm{T}}\left[\boldsymbol{\tau} + \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z})\right] + \kappa\varsigma_1 + \kappa\varsigma_2 + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\left[\boldsymbol{\Gamma}\boldsymbol{\varphi}(\mathbf{Z})\mathbf{z}_2^{\mathrm{T}} - \dot{\hat{\boldsymbol{\theta}}}\right] + \omega_{1,0}\mathbf{z}_2\tanh\left(\frac{\omega_{1,0}\mathbf{z}_2}{\varsigma_2}\right)\end{aligned} \tag{24}$$

Design the adaptive tracking control law $\boldsymbol{\tau}$ and adaptive law $\dot{\hat{\boldsymbol{\theta}}}$ as

$$\begin{aligned}\boldsymbol{\tau} = & -\frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_2^{1+\beta} - \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}(\mathbf{Z}) - \mathbf{P}\mathbf{R}(\psi)\mathbf{z}_2 - \frac{\pi}{\beta\mathrm{T}_{\max}}\mathbf{z}_2^{1-\beta} \\ & \times\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\tanh\left(\frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\frac{\mathbf{z}_2^{2-\beta}}{\varsigma_2}\right) - \omega_{1,0}\tanh\left(\frac{\mathbf{z}_2\omega_{1,0}}{\varsigma_2}\right)\end{aligned} \tag{25}$$

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\left(\boldsymbol{\varphi}(\mathbf{Z})\mathbf{z}_2^{\mathrm{T}} - \mathrm{K}\hat{\boldsymbol{\theta}}\right) \tag{26}$$

where $\omega_{1,0}$ will be defined later, $\varsigma_2 > 0$ is a constant. $\mathrm{K} > 0$ is a constant.

Thus, invoking (24)-(26), $\dot{V}_2$ can be finally written as

$$\begin{aligned}\dot{V}_2 \le & -\frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{2+\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta} + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\hat{\boldsymbol{\theta}} \\ & + \kappa\varsigma_1 + 2\kappa\varsigma_2 - \frac{2^\beta\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_2^{2+\beta} - \frac{\pi}{\beta\mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_2^{2-\beta}\end{aligned} \tag{27}$$

3.2. **Stability analysis.** The following theorem is summarized to illustrate the properties of the above developed adaptive predefined-time constraint tracking control law for AUVs.

Theorem 3.1. *For AUV system (1), Assumptions 2.1 and 2.2 hold, if we adopt the control law (25), virtual control law (13), adaptive law (26); thus, the proposed control algorithm has the following properties:*

1) AUV can track the desired trajectory $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^{\mathrm{T}}$ *in predefined time;*

2) All closed-loop signals are bounded.

Proof: Choose the whole Lyapunov as

$$V = \frac{1}{2}\mathbf{z}_1^{\mathrm{T}}\mathbf{z}_1 + \frac{1}{2}\mathbf{z}_2^{\mathrm{T}}\mathbf{M}\mathbf{z}_2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} \tag{28}$$

From (16), (27) and (28), $\dot{V}$ is

$$\begin{aligned}\dot{V} \leq & -\frac{2^\beta\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_1^{2+\beta} - \frac{\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_1^{2-\beta} + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathrm{K}\hat{\boldsymbol{\theta}} \\ & + \kappa\varsigma_1 + 2\kappa\varsigma_2 - \frac{2^\beta\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1+\frac{\beta}{2}}\mathbf{z}_2^{2+\beta} - \frac{\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\right)^{1-\frac{\beta}{2}}\mathbf{z}_2^{2-\beta}\end{aligned} \tag{29}$$

Similar to (12), the following inequality holds

$$\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathrm{K}\hat{\boldsymbol{\theta}} \leq \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathrm{K}\tilde{\boldsymbol{\theta}} + \frac{1}{2}\boldsymbol{\theta}^{*\mathrm{T}}\mathrm{K}\boldsymbol{\theta}^* \tag{30}$$

From Lemma 2.1, assume $\left\|\tilde{\boldsymbol{\theta}}\right\| \leq \delta^*$; thus, we have

$$-\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathrm{K}\tilde{\boldsymbol{\theta}} \leq -\left(\frac{\mathrm{K}}{2}\left\|\tilde{\boldsymbol{\theta}}\right\|\right)^{1-\frac{\beta}{2}} - \left(\frac{\mathrm{K}}{2}\left\|\tilde{\boldsymbol{\theta}}\right\|\right)^{1+\frac{\beta}{2}} + \overline{\Gamma} \tag{31}$$

where $\overline{\Gamma} = \frac{\beta}{2}\left(\frac{2-\beta}{2}\right)^{\frac{2-\beta}{\beta}} + \left[\frac{\delta^*}{2}\mathrm{K}\right]^{1+\frac{\beta}{2}}$.

Thus, from (29)-(31), $\dot{V}$ can be further rewritten as

$$\begin{aligned}\dot{V} \leq & -\frac{2^\beta\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\mathbf{z}_1^2\right)^{1+\frac{\beta}{2}} - \frac{\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\mathbf{z}_1^2\right)^{1-\frac{\beta}{2}} - \frac{2^\beta\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\mathbf{z}_2^2\right)^{1+\frac{\beta}{2}} \\ & -\frac{\pi}{\beta \mathrm{T}_{\max}}\left(\frac{1}{2}\mathbf{z}_2^2\right)^{1-\frac{\beta}{2}} - \mathrm{K}^{1-\frac{\beta}{2}}\lambda_{\max}(\boldsymbol{\Gamma}^{-1})\left(\frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\right)^{1-\frac{\beta}{2}} + \overline{\Gamma} \\ & - \mathrm{K}^{1+\frac{\beta}{2}}\lambda_{\max}(\boldsymbol{\Gamma}^{-1})\left(\frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\right)^{1+\frac{\beta}{2}} + \kappa\varsigma_1 + 2\kappa\varsigma_2 + \frac{1}{2}\boldsymbol{\theta}^{*\mathrm{T}}\mathrm{K}\boldsymbol{\theta}^*\end{aligned} \tag{32}$$

With the help of Lemma 2.2, choose parameters ϵ and $\bar{\epsilon}$ as $\epsilon = \min\Big\{1, \left(\lambda_{\min}(\mathbf{K})\mathrm{K}\lambda_{\max}(\boldsymbol{\Gamma}^{-1})\right)^{1-\frac{\beta}{2}}\Big\}$, $\bar{\epsilon} = \min\Big\{2^\beta, \left(\lambda_{\min}(\mathbf{K})\mathrm{K}\lambda_{\max}(\boldsymbol{\Gamma}^{-1})\right)^{1+\frac{\beta}{2}}\Big\}$, and $\dot{V}$ can be further rewritten as

$$\dot{V} \leq -\frac{\epsilon\pi}{\beta \mathrm{T}_{\max}}V_1^{1-\frac{\beta}{2}} - \frac{\bar{\epsilon}\pi}{\beta \mathrm{T}_{\max}}V_1^{1+\frac{\beta}{2}} + D \tag{33}$$

where $D = \kappa\varsigma_1 + 2\kappa\varsigma_2 + \frac{1}{2}\boldsymbol{\theta}^{*\mathrm{T}}\mathrm{K}\boldsymbol{\theta}^* + \overline{\Gamma}$.

To achieve the predefined-time control objective, inspired by [11], define a set as

$$\Omega = \left\{V \middle| V \leq \min\left\{\left[\frac{2\beta D\mathrm{T}_{\max}}{\epsilon\pi}\right]^{\frac{2}{2-\beta}}, \left[\frac{2\beta D\mathrm{T}_{\max}}{\bar{\epsilon}\pi}\right]^{\frac{2}{2+\beta}}\right\}\right\} \tag{34}$$

thus, we know V can converge into the set Ω in predefined time $\overline{\mathrm{T}} \leq \sqrt{2}\mathrm{T}_{\max}$. Obviously, from the definition of V, we know tracking and virtual errors $\mathbf{z}_1$ and $\mathbf{z}_2$, estimation error $\tilde{\boldsymbol{\theta}}$, and other closed-loop signals are all bounded in predefined time $\sqrt{2}\mathrm{T}_{\max}$. The proof of Theorem 3.1 is thus completed. □

4. **Conclusions.** We have investigated the issue of predefined-time trajectory tracking adaptive constraint control for underactuated AUVs. The ideal trajectory tracking accuracy of AUVs has been ensured by adopting time-varying asymmetric barrier functions. Then, under frame of adaptive backstepping control, a novel predefined-time trajectory tracking control approach has been developed. Based on predefined-time stable theory, the developed control method can ensure the AUVs' trajectory tracking error converges into a preset error region, and all closed-loop signals are bounded in predefined time. Future research direction will extend the proposed control method of this paper to multi-agent systems.

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