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A DESIGN METHOD OF CONTROL SYSTEM CONSIDERING A DETERIORATION OF THE PLANT

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Abstract. *In this paper, we examine a design method for control system considering a deterioration of the plant for a single-input/single-output minimum-phase biproper plant. In a practical control system design, we need to consider making the control system stable for a structural change of plant and/or faults of plants such as loss of sensors, and actuators. In order to overcome this problem, we present the parameterization of all stable stabilizing controllers for the plant with the fault and deterioration. In addition, we propose a design method of control system for the plant with the fault and deterioration.*
Keywords: The parameterization problem, Fault tolerant control, A stability for a deterioration of the plant, Minimum phase system

1. **Introduction.** Several automated systems often have an event of failure of system components such as actuators, and sensors. The fault and failure of system components often make the system vulnerable; the fault and failure often make the control system unstable, and/or lead to lack of several performances of control system such as tracking characteristics [1]. This is particularly important for safety-critical systems, such as aircraft, spacecraft, nuclear power plants, and chemical plants processing hazardous materials. In such systems, the consequences of a minor fault in a system component can be catastrophic [2]. The control for the plant with faults such as safety-critical systems is required to make a control system have a fault tolerance in the meaning of keeping stability and performances of the system for faults. This control is referred to as the falt-tolerant control [3]. The fault tolerant control is studied by several researchers [1, 2, 3, 4]. Neumann provides a concept of the fault tolerance, which is to construct a more reliable system through depulication by using less reliable components [5]. Owens considers the fault tolerant control system as a control system which prossesses integrity or which has control loops which prosses loop integrity [6]. In [7, 8], the fault tolerant control system is considered as the reliable control system. Stengel points out that the fault tolerant control sometimes is equaivalent to reliable control but there are some differences, and gives definition of reliability [9]. Blanke et al. demonstrate the principle involved in the systematic design and development of a real fault tolerant control application [10]. According to [11], the main task of the fault tolerant control is the design of a controller with suitable structure to guarantee stability and satisfactory performance, not only when all control components are operational, but also in the case when sensors, actuators malfunction.

According to [1, 2, 4, 12, 13, 14, 15], fault tolerant control systems can be categorized into two main classes: active fault tolerant control system and passive fault tolerant control system. Active fault tolerant control system reacts to each fault differently [16]. The

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reaction of the active fault tolerant control system to faults occurs by the information received from the detection system in the control system. The active fault tolerant control system is constructed by three components: a fault diagnosis system to detect faults, a fault compensator system to supervise faults, and a control system. If a fault detection by fault diagnosis is accurate, the active fault tolerant control system is robust against the imperfect fault detection information, and the time for fault recovery is less than the available time for recovery, the active fault tolerant control system maintains the stability and satisfactory performance of the system for faults. Some studies consider a design method of the active fault tolerant control system from several viewpoints of sliding mode control [17, 18, 19, 20], robust control [22, 23], predictive control [24], linear quadratic control [25], fuzzy logic control [26, 27], adaptive control [28, 29], Lyapunov-based control [30, 31], and so on. The design method of active fault tolerant control system considers to apply to several applications: near space hypersonic vehicle [27], spacecraft [32], electric vehicles [33], flight control system [20, 21, 26, 34], and so on.

However, the active fault tolerant control system has following disadvantages: 1) since several steps such as fault detection, and supervision can be based on excessive computations and are needed to take lots of time, active fault tolerant control systems could be its slow response; 2) since active fault tolerant control system is sensitive to the result obtained from fault detection and isolation, it is difficult to design the control system for nonlinear system with uncertainties [35]. These disadvanteges often lead to reducing performances of the control system in the meaning of the tracking performance, disturbance attenuation, and so on, for the known faults and failure such as deterioration of the plant.

In contrast to the active fault tolerant control system, the passive fault tolerant control system is designed from the view point, which has redundancy of the control system and robust stability to possible system faults [27]. The passive fault tolerant control system is designed to make the system robustly stable for a set of model of plants. The studies of the passive fault tolerant control system are started from [8, 36, 38]. Vidyasagar and Viswanadham consider a problem to stabilize the control system for some plants, which describe a nominal plant and represent plants changed by faults, by a controller [36]. Vidyasagar and Viswanadham clarify this problem to be attributed to the strongly stabilization problem [36, 37]. The strong stabilization problem is a problem to make the control system stable by using a stable controller. By using results of [36, 38], the passive fault tolerlant control system can be constructed by solving the simultaneous stabilization problem by the plant if a nominal plant and all plants, which represent plants changed by faults, are given. However, the necessary and sufficient condition of the simultaneous stabilization problem for three or more plants is not clarified [38]. The result of [38] is difficult to apply to the system with faults what is described as three or more plants. In order to make a control system stable for the plant with faults, it is required to describe the plant with faults as one plant [38]. One of modifications in this paper is to provide a new description of the plant presenting faults due to a design method of the passive fault tolerant control system.

If a plant with a failure process is described, then we could systematically obtain all stabilizing controller for a control system with a fault tolerance for its plant. A problem to obtain all stabilizing controllers for the plant is called the parameterization problem [38, 39, 40, 41, 42]. For a stable plant, the parameterization of all stabilizing controllers has a structure identical to that of internal model control [40]. Glaria and Goodwin derive a parametrization of all internally stabilizing controllers for single-input/single-output minimum phase system [41]. However, the result of [41] remains a problem that the parameterization of all internally stabilizing controllers given by the result of [41] includes improper controllers. Since the controller is required to be proper, the result of [41] cannot use practical applications. Yamada and Moki consider these problems, and give the parameterization of all proper internally stabilizing controllers for single-input/single-output minimum phase plants [42, 43]. By using the results of [42, 43], we can design the internally stabilizing controller for the single-input/single-output minimum phase system.

If all stabilizing controllers for two plants, which are the normal plant and the plant presenting the faults and/or deterioration, are obtained, we could allow the systematic design of a control system for the plant able to cause a fault and/or deterioration. One of motivations is to obtain a parameterization of all stabilizing controllers to make the control system stable, for the nominal plant and plant presenting, simultaneously.

In this paper, we consider the problem to obtain the parameterization of all the stabilizing controllers for the plant with the fault and deterioration. In addition, we propose a design method for control system for the plant with the fault and deterioration. This paper is organized as follows. In Section 2, we formulate the problem considered in this paper. In Section 3, we present the parameterization of all stable stabilizing controllers for the plant with the fault and deterioration. In Section 4, we propose a design method of control system for the plant with the fault and deterioration. In Section 5, a numerical example is shown to illustrate the effectiveness of the proposed method. Section 6 gives concluding remarks.

Notations

$\mathbf{R}$	the set of real numbers.
$[a, b]$	the closed interval defined as $[a, b] := \{x \in \mathbf{R} \mid a \le x \le b, a \in \mathbf{R}, b \in \mathbf{R}\}$
(a, b)	the open interval defined as $(a, b) := \{x \in \mathbf{R} \mid a < x < b, a \in \mathbf{R}, b \in \mathbf{R}\}$
$\mathbf{C}$	the set of complex numbers.
$R(s)$	the set of real relational functions with s.
RH_∞	the set of stable proper real relational functions.
$\mathcal{U}$	the set of unimodular function with s, that is, $\mathcal{U} := \{A(s) \in RH_\infty \mid A^{-1} \in RH_\infty\}$
$\Re\{\cdot\}$	real part of $\cdot$.
$\delta(\cdot)$	the relative degree of $\cdot$

2. **Problem Formulation.** Consider the control system described as

$$\begin{cases} y(s) = G(s)u(s) \\ u(s) = C(s)(r(s) - y(s)) + d(s) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s)$ is a controller, $r(s) \in R(s)$ is the reference input, $y(s) \in R(s)$ is the output, and $d(s) \in R(s)$ is the disturbance. The plant $G(s) \in R(s)$ is assumed to be of minimum phase and biproper. The plant $G(s)$ is denoted by

$$G(s) = G_1(s) \left(1 - \tau G_2(s)\right), \tag{2}$$

where $G_1(s) \in R(s)$ is of minimum phase and biproper, $G_2(s) \in \mathcal{U}$ satisfies

$$|G_2(j\omega)| < 1 \ (\forall \omega \in R), \tag{3}$$

and $\tau \in \mathbf{R}$ is an arbitrary real number included in $[0, 1]$. $G_1(s)$ means the plant before affecting a fault, deterioration, and so on. On the other hand, $G_2(s)$ means the element of perturbation adding to $G_1(s)$ by the fault, deterioration, and so on. τ in (2) means a changing rate of the plant by affecting the fault, deterioration, and so on. $C(s) \in R(s)$ is the controller to stabilize $G_1(s) \in R(s)$.

The problem considered in this paper is to propose a design method of control system in (1).

3. **Parameterization of All Stabilizing Controllers.** In this section, we clarify the parameterization of all stabilizing controllers $C(s)$ for the plant $G(s)$ in (2).

According to [41, 43], the parameterization of all stabilizing controllers $C(s)$ for the plant $G(s)$ in (2) is summarized as Theorem 3.1.

Theorem 3.1. [41, 43] *The control system in (1) is internally stable if and only if $C(s)$ is written by*

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G_1(s)(1 - \tau G_2(s))}, \tag{4}$$

where $Q(s)$ is any minimum-phase and biproper rational function.

Proof: Proof is obvious from Theorem 1 in [43]. □

Note that stabilizing controller $C(s)$ in (4) is related to τ. Therefore, when τ in (4) is changed, in some cases, the control system in (1) is unstable. If the controller $C(s)$ stabilizes the control system in (1) independent from τ, it is useful. In the next section, we examine a design method for control system in (1) independent from τ.

4. **Controller Design Independent from $\boldsymbol{\tau}$.** In this section, we examine a design method of stabilizing controller $C(s)$ independent from τ. From (2) and (3), there exists $W(s) \in RH_\infty$ satisfying

$$\left| \frac{\tau G_2(j\omega)}{1 - \tau G_2(j\omega)} \right| \le |W(j\omega)| \quad \forall \omega \in R. \tag{5}$$

By using $W(s)$ satisfying (5), we clarify stability condition, which is independent from τ. The stability condition is summarized as the following theorem.

Theorem 4.1. *Assume that $C(s)$ stabilizes $G_1(s)$. The control system in (1) is internally stable, if $C(s)$ given by*

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G_1(s)} \tag{6}$$

satisfies

$$\left\| \frac{Q(s)}{G_1(s)} W(s) \right\|_\infty < 1, \tag{7}$$

where $Q(s)$ is minimum phase and biproper rational function.

To prove Theorem 4.1, a necessary lemma is shown.

Lemma 4.1. *Assume that $G(s)$ has no zeros in the closed right half plane and the p-th number of pole in the closed right half plane, $G_1(s)$ has no zeros in the closed right half plane and the p_1-th number of pole in the closed right half plane. The Nyquist plot of $1 - \tau G_2(s)$ for $-\infty \le \omega \le \infty$ encircles the origin $(0, 0)$ $p - p_1$ times in the counter-clockwise direction.*

The proof of Lemma 4.1 is shown as follows.

Proof: From the assumption that $G(s)$ and $G_1(s)$ are biproper and of minimum phase, $G(s)$ and $G_1(s)$ are denoted by

$$G(s) = \frac{k \prod_{i=1}^{n} (s + \mu_i)}{\prod_{i=1}^{p} (s - \gamma_i) \prod_{i=p+1}^{n} (s + \gamma_i)}, \tag{8}$$

and

$$G_1(s) = \frac{k_1 \prod_{i=1}^{n_1} (s + \mu_{1i})}{\prod_{i=1}^{p_1} (s - \gamma_{1i}) \prod_{i=p_1+1}^{n_1} (s + \gamma_{1i})}, \tag{9}$$

respectively, where $\Re\{\gamma_i\} > 0$ $(i = 1, 2, \ldots, n)$, $\Re\{\gamma_{1i}\} > 0$ $(i = 1, 2, \ldots, n_1)$, $\Re\{\mu_i\} > 0$ $(i = 1, 2, \ldots, n)$ and $\Re\{\mu_{1i}\} > 0$ $(i = 1, 2, \ldots, n_1)$. From (2), (8) and (9), $1 - \tau G_2(s)$ is written by

$$1 - \tau G_2(s) = \frac{G(s)}{G_1(s)} = \frac{k \prod_{i=1}^{n} (s + \mu_i)}{\prod_{i=1}^{p} (s - \gamma_i) \prod_{i=p+1}^{n} (s + \gamma_i)} \frac{\prod_{i=1}^{p_1} (s - \gamma_{1i}) \prod_{i=p_1+1}^{n_1} (s + \gamma_{1i})}{k_1 \prod_{i=1}^{n_1} (s + \mu_{1i})}. \tag{10}$$

This yields

$$\angle(1-\tau G_2(s)) = \sum_{i=1}^{n}\angle(s+\mu_i) + \sum_{i=1}^{p_1}\angle(s-\gamma_{1i}) + \sum_{i=p_1+1}^{n_1}\angle(s+\gamma_{1i}) - \sum_{i=1}^{p}\angle(s-\gamma_i)$$
$$- \sum_{i=p+1}^{n}\angle(s+\gamma_i) - \sum_{i=1}^{n_1}\angle(s+\mu_{1i}). \tag{11}$$

From (11) and Cauchy's argument principle, the Nyquist plot of $1-\tau G_2(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ $p-p_1$ times in the counter-clockwise direction. □

In this way, the proof of Lemma 4.1 has been shown.

Using Lemma 4.1, we will prove Theorem 4.1.

Proof: The characteristic polynomial of the control system in (1) is given by $1+G(s)C(s)$. If the Nyquist plot of $1+G(s)C(s)$ for $-\infty < \omega < \infty$ encircles the origin $(0,0)$ $p-z_c$ times in the counter-clockwise direction, then the control system in (1) is stable. Here, z_c denotes the number of zeroes of $C(s)$ in the closed right half plane. From simple manipulation, the characteristics polynomial is rewritten by

$$1+G(s)C(s) = (1+G_1(s)C(s))(1-\tau G_2(s))\left(1+\frac{1}{1+G_1(s)C(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)}\right) \tag{12}$$

$$C(s) = \frac{k_c\prod_{i=1}^{z_c}(s-\mu_{ci})\prod_{i=z_c+1}^{n_c}(s+\mu_{ci})}{\prod_{i=1}^{n_c}(s+\gamma_{ci})} \tag{13}$$

$$\begin{aligned}&1+G(s)C(s)\\ &= 1+\frac{k\prod_{i=1}^{n}(s+\mu_i)}{\prod_{i=1}^{p}(s-\gamma_i)\prod_{i=p+1}^{n}(s+\gamma_i)}\frac{k_c\prod_{i=1}^{z_c}(s-\mu_{ci})\prod_{i=z_c+1}^{n_c}(s+\mu_{ci})}{\prod_{i=1}^{n_c}(s+\gamma_{ci})}\end{aligned} \tag{14}$$

$$\begin{aligned}&1+G_1(s)C(s)\\ &= 1+\frac{k\prod_{i=1}^{n1}(s+\mu_{1i})}{\prod_{i=1}^{p_1}(s-\gamma_{1i})\prod_{i=p_1+1}^{n1}(s+\gamma_{1i})}\frac{k_c\prod_{i=1}^{z_c}(s-\mu_{ci})\prod_{i=z_c+1}^{n_c}(s+\mu_{ci})}{\prod_{i=1}^{n_c}(s+\gamma_{ci})}\end{aligned} \tag{15}$$

From the assumption that $C(s)$ stabilizes $G_1(s)$, the Nyquist plot of $1+G_1(s)C(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ p_1-z_c times in the counter-clockwise direction. This means that if the Nyquist plot of

$$(1-\tau G_2(s))\left(1+\frac{1}{1+G_1(s)C(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)}\right)$$

encircles the origin $p-p_1$ times in the counter-clockwise direction for any $\tau \in [0,1]$, then the control system in (1) is internally stable. From Lemma 4.1, the Nyquist plot of $1-\tau G_2(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $p-p_1$ times in the counter-clockwise direction. Therefore, the control system in (1) is internally stable if the Nyquist plot of

$$1+\frac{1}{1+G_1(s)C(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)} \tag{16}$$

for $-\infty \leq \omega \leq \infty$ does not encircle the origin $(0,0)$ any times. Substituting (6) to (16), we have

$$1+\frac{1}{1+G_1(s)C(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)} = 1+\frac{Q(s)}{G_1(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)}. \tag{17}$$

From the assumption that $Q(s)$ satisfies (7), it is obvious that the Nyquist plot of

$$1+\frac{Q(s)}{G_1(s)}\frac{\tau G_2(s)}{1-\tau G_2(s)}$$

for $-\infty \leq \omega \leq \infty$ does not encircle the origin $(0,0)$ any times for any $\tau \in [0,1]$.

From above discussion, Theorem 4.1 has been proven. □

5. **Numerical Example.** In this section, we show a numerical example to illustrate features of the proposed method.

Consider the problem to design the control system in (1) for $G(s)$ in (2). Here, $G_1(s)$ and $G_2(s)$ in (2) are given by

$$G_1(s) = \frac{s+1}{s-2} \tag{18}$$

and

$$G_2(s) = \frac{0.09s+1}{s+2}, \tag{19}$$

respectively.

The gain plot of $G_2(s)$ is shown in Figure 1. Here, the solid line shows the gain plot of $G_2(s)$. Figure 1 shows that $G_2(s)$ satisfies $|G_2(j\omega)| < 1$ $(\forall\omega \in R)$. From Figure 1, $W(s)$ satisfying (5) is designed by

$$W(s) = \frac{s+10}{s+1}. \tag{20}$$

In order to confirm that $W(s)$ in (20) satisfies (5), we show the gain plot of $W(s)$ and $\tau G_2(s)/(1-\tau G_2(s))$ as Figure 2. Here, the solid line shows the gain plot of $W(s)$, and broken lines show the gain of $\tau G_2(s)/(1-\tau G_2(s))$ for $\tau = 0, 0.01, 0.02, \ldots, 1.0$. Figure 2 shows that $W(s)$ in (20) satisfies (5).

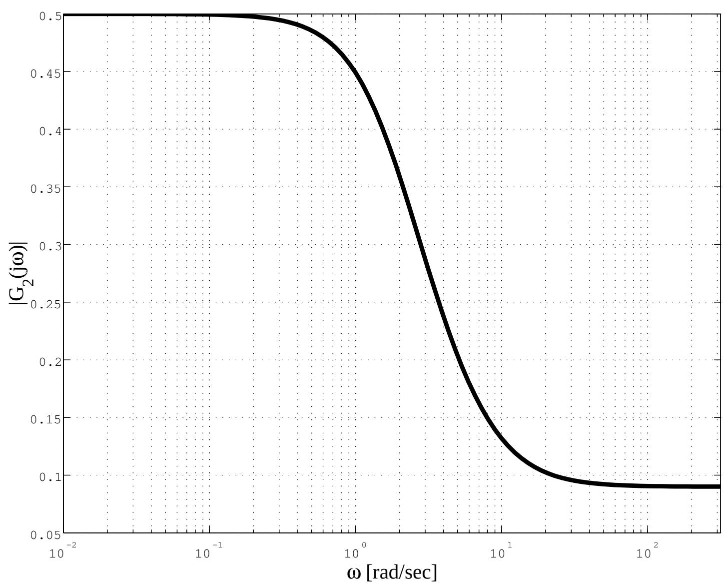

Figure 1. The gain plot of $G_2(s)$

$Q(s)$ in (6) satisfying (7) is settled as

$$Q(s) = \frac{s(s+1)^2}{(s+10)^2(s+2)}. \tag{21}$$

In order to confirm that $Q(s)$ in (21) satisfies (7), we show the gain of $1/W(s)$ and $Q(s)/G_1(s)$ shown in Figure 3. Here, the solid line shows the gain plot of $Q(s)/G_1(s)$ and the broken line shows that of $1/W(s)$. From Figure 3, $Q(s)$ satisfies (7).

Using $Q(s)$ in (21), the controller is given by (6) and obtained as

$$C(s) = \frac{-0.000024189(s-950800)(s+4)(s+2.174)(s+1)}{s(s+1)^3}. \tag{22}$$

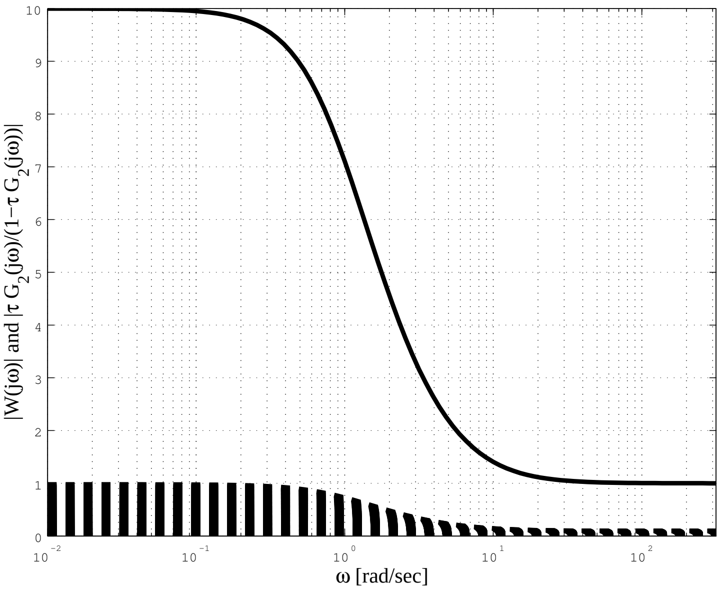

FIGURE 2. The gain plot of $W(s)$ and $\tau G_2(s)/(1-\tau G_2(s))$

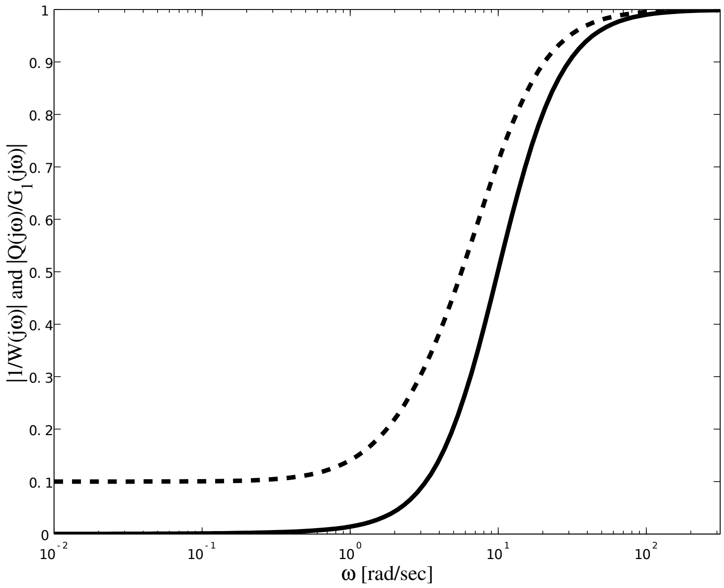

FIGURE 3. The gain plot of $Q(s)/G_1(s)$ and $1/W(s)$

When the reference input $r(t)$ is given by

$$r(t) = 1, \tag{23}$$

the response of the output $y(t)$ for $G(s)$ in (2), in the cases of $\tau = 0, 0.01, 0.02, \ldots, 1.0$ is shown in Figure 4.

Figure 4 shows the control system in (1) is stable independent from τ.

When the disturbance $d(t)$ is given by

$$d(t) = 1, \tag{24}$$

the response of the output $y(t)$ for the disturbance $d(t)$ is shown in Figure 5.

From Figure 5, the control system in (1) is stable independent from τ. Figure 4 and Figure 5 show that for $G(s)$ in (2), the proposed method is effective to make the control system in (1) internally stable.

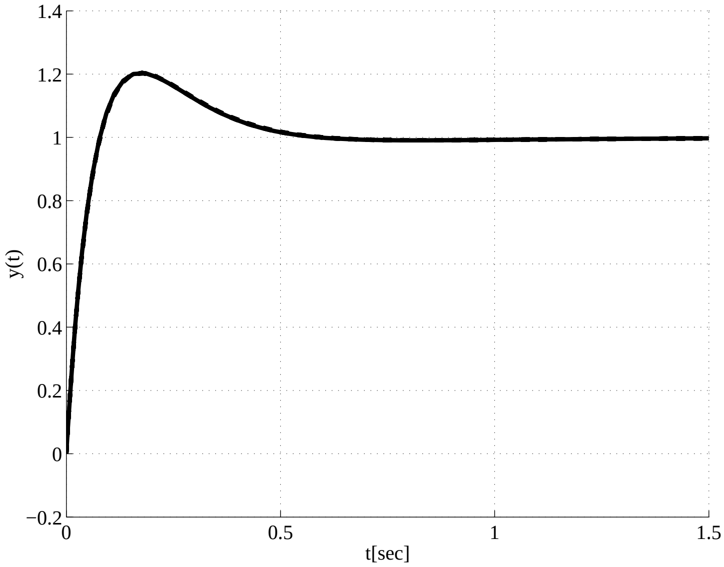

FIGURE 4. The response of the output $y(t)$ for the reference input $r(t)$ in (23), in the cases of $\tau = 0, 0.01, 0.02, \ldots, 1.0$

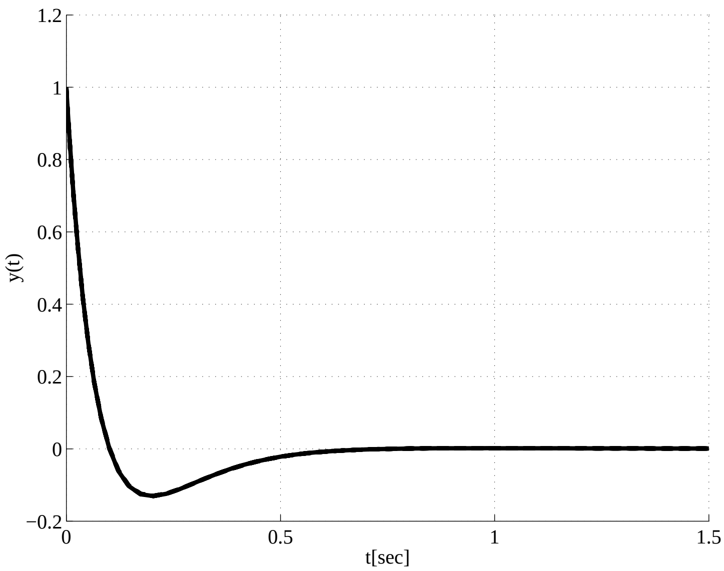

FIGURE 5. The response of the output $y(t)$ for the disturbance $d(t)$ in the cases of $\tau = 0, 0.01, 0.02, \ldots, 1.0$

6. **Conclusion.** In this paper, we have considered a parameterization of the all stabilizing controllers for the plant with the fault and deterioration for a single-input/single-output minimum phase system considering deterioration. To obtain this parameterization of all stabilizing controllers for plant with fault and deterioration, we have proposed a new description of its plant. It has been clarified that by using the proposed description of the plant, the parameterization of all stabilizing controllers for the plant with fault and deterioration is obtained. Furthermore, to illustrate feature of the proposed method, we have shown a numerical example. However, a design procedure of a controller to satisfy the proposed parameterization is not clarified. In addition, an application of the result of this paper is also not clarified. These will be explained in another papers. As for applications of the results of this paper, it will be explained in future papers.

REFERENCES

[1] M. Blanke, M. Kinnaert, J. Lunze and M. Stariswiecki, *Diagnosis and Fault-Tolerant Control*, 3rd Edition, Springer, Heidelberg, 2016.
[2] Y. Zhang and J. Jiang, Bibliographical review on reconfigurable fault-tolerant control systems, *Annual Reviews in Control*, vol.32, no.2, pp.229-252, 2008.
[3] H. Noura, D. Theilliol, J. C. Ponsart and A. Chamseddine, *Fault-tolerant Control System Design and Practical Applications*, Springer, 2009.
[4] X. Yu and J. Jiang, A survey of fault-tolerant controllers based on safety-related issues, *Annual Reviews in Control*, vol.39, pp.46-57, 2015.
[5] J. V. Neumann, Probabilistic logics and synthesis of reliable organisms, *Annals of Mathematics Studies*, vol.34, no.1, pp.43-98, 1956.
[6] D. H. Owens, *Feedback and Multivariable Systems*, Peregrinus, 1978.
[7] J. D. Birdwell, D. A. Castanon and M. Athans, On reliable control system designs, *IEEE Transactions on Systems, Man, and Cybernetics*, vol.16, no.5, pp.703-711, 1986.
[8] R. J. Veillette, J. B. Medanic and W. R. Perkins, Design of reliable control systems, *IEEE Transactions on Automatic Control*, vol.37, no.3, pp.290-304, 1992.
[9] R. F. Stengel, Toward intelligent flight control, *IEEE Transactions on Systems, Man, and Cybernetics*, vol.23, no.6, pp.1699-1717, 1993.
[10] M. Blanke, R. Izadi-Zamanabadi, R. Bogh and Z. P. Lunau, Fault-tolerant control systems – A holistic view, *Control Engineering Practice*, vol.5, no.5, pp.693-702, 1997.
[11] R. J. Patton, Fault-tolerant control: The 1997 situation, *IFAC Proceedings Volumes*, vol.30, no.18, pp.1029-1051, 1997.
[12] H. Alwi, C. Edwards and C. P. Tan, Fault tolerant control and fault detection and isolation, in *Fault Detection and Fault-Tolerant Control Using Sliding Modes*, London, Springer, 2011.
[13] J. Jiang and X. Yu, Fault-tolerant control systems: A comparative study between active and passive approaches, *Annual Reviews in Control*, vol.36, no.1, pp.60-72, 2012.
[14] X. Yu and J. Jiang, A survey of fault-tolerant controllers based on safety-related issues, *Annual Reviews in Control*, vol.39, pp.46-57, 2015.
[15] X. Qi, J. Qi, D. Theilliol, Y. Zhang, J. Han, D. Song and C. Hua, A review on fault diagnosis and fault tolerant control methods for single-rotor aerial vehicles, *Journal of Intelligent & Robotic Systems*, vol.73, pp.535-555, 2014.
[16] A. Abbaspour, S. Mokhtari, A. Sargolzaei and K. K. Yen, A survery on active fault-tolerant control systems, *Electronics*, vol.9, no.9, 1513, 2020.
[17] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Kluwer Academic, Boston, 1999.
[18] Z. Yang, M. Blanke and M. Verhaegen, Robust control mixer method for reconfigurable control design using model matching, *IET Control Theory & Applications*, vol.1, pp.347-357, 2007.
[19] H. Shen, J. H. Park and Z. G. Wu, Finite-time reliable L_2-L_∞/H_∞ control for Takagi-Sugeno fuzzy systems with actuator faults, *IET Control Theory & Applications*, vol.1, 2014.
[20] B. Wang, Y. Shen and Y. Zhang, Active fault-tolerant control for a quadrotor helicopter against actuator faults and model uncertainties, *Aerospace Science and Technology*, vol.99, 2020.
[21] R. Singh and B. Bhushan, Data-driven technique-based fault-tolerant control for pitch and yaw motion in unmanned helicopters, *IEEE Transactions on Instrumentation and Measurement*, vol.70, pp.1-11, 2020.
[22] D. V. Nair and M. S. R. Murty, Reconfigurable control as actuator fault-tolerant control design for power oscillation damping, *Protection and Control of Modern Power Systems*, vol.5, pp.1-12, 2020.
[23] P. Selvaraj, O. M. Kwon, S. H. Lee and R. Sakthivel, Robust fault-tolerant control design for polynomial fuzzy systems, *Fuzzy Sets and Systems*, 2022.
[24] K. Patan, *Robust and Fault-Tolerant Control: Neural-Network-Based Solutions*, Springer, Heidelberg, 2018.
[25] S. Sun, X. Wang, Q. Chu and C. de Visser, Incremental nonlinear fault-tolerant control of a quadrotor with complete loss of two opposing rotors, *IEEE Transactions on Robotics*, vol.37, no.1, pp.116-130, 2020.
[26] H. R. Patel and V. A. Shah, General type-2 fuzzy logic systems using shadowed sets: A new paradigm towards fault-tolerant control, *Australian & New Zealand Control Conference (ANZCC)*, pp.116-121, 2021.
[27] Q. Shen, B. Jiang and P. Shi, *Fault Diagnosis and Fault-Tolerant Control Based on Adaptive Control Approach*, Springer, Heidelberg, 2017.

[28] F. Gao and J. Liu, Partial differential equation modeling and vibration control for a nonlinear 3D rigid-flexible manipulator system with actuator faults, *International Journal of Robust and Nonlinear Control*, vol.29, no.11, 2019.

[29] Z. Zhao, Z. Liu, W. He, K. S. Hong and H. X. Li, Boundary adaptive fault-tolerant control for a flexible Timoshenko arm with backlash-like hysteresis, *Automatica*, vol.130, 109690, 2020.

[30] M. Benosman and K. Y. Lum, Passive actuators' fault-tolerant control for affine nonlinear systems, *IEEE Transactions on Control Systems Technology*, vol.18, no.1, pp.152-163, 2010.

[31] R. de Castro and J. Brembeck, Lyapunov-based fault tolerant control allocation, *Vehicle System Dynamics*, vol.60, no.10, pp.3605-3630, 2022.

[32] Q. Hu, Y. Zhang, H. Xing and X. Bing, Adaptive integral-type sliding mode control for spacecraft attitude maneuvering under actuator stuck failures, *Chinese Journal of Aeronautics*, vol.24, pp.32-45, 2011.

[33] R. Wang and J. Wang, Passive actuator fault-tolerant control for a class of overactuated nonlinear systems and applications to electric vehicles, *IEEE Transactions on Vehicular Technology*, vol.62, no.3, pp.972-985, 2013.

[34] P. Yang, Z. Wang, Z. Zhang and X. Hu, Sliding mode fault tolerant control for a quadrotor with varying load and actuator fault, *Actuators*, vol.10, no.12, 323, 2021.

[35] A. A. Amin and K. M. Hasan, A review of fault tolerant control systems: Advancements and applications, *Measurement*, vol.143, pp.58-68, 2019.

[36] M. Vidyasagar and N. Viswanadham, Algebraic design techniques for reliable stabilization, *IEEE Transactions on Automatic Control*, vol.27, no.5, pp.1085-1095, 1982.

[37] R. Saeks and J. Murray, Fractional representation, algebraic geometry, and the simultaneous stabilization problem, *IEEE Transactions on Automatic Control*, vol.27, no.4, pp.895-903, 1982.

[38] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, MIT Press Cambridge, MA, 1985.

[39] D. Youla, J. Bongiorno and H. Jabr, Modern Wiener-Hopf design of optimal controllers Part I: The single-input-output case, *IEEE Transactions on Automatic Control*, vol.21, no.1, pp.3-13, 1976.

[40] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice-Hall, 1989.

[41] J. J. Glaria and G. C. Goodwin, A parameterization for the class of all stabilizing controllers for linear minimum phase plants, *IEEE Transactions on Automatic Control*, vol.39, no.2, pp.433-434, 1994.

[42] K. Yamada, A parametrization for the class of all proper stabilizing controllers for the cartain class of systems and their control tructure, *IFAC Proceedings Volumes*, vol.34, no.13, pp.207-212, 2001.

[43] K. Yamada and T. Moki, A study on control design method using parallel compensation technique, *Transactions of the Japan Society of Mechanical Engineers*, vol.69, no.687, pp.3015-3020, 2003 (in Japanese).