ICIC Express Letters
Volume **18**, Number **4**, April **2024**

ICIC International ©2024 ISSN 1881-803X
pp. 343–352

A NEW APPROACH OF NEUTROSOPHIC SETS APPLIED ON BIPOLAR FUZZY IDEAL IN SEMIGROUPS

Natthinee Deetae[1] and Pannawit Khamrot[2,*]

[1]Department of Statistics
Faculty of Science and Technology
Pibulsongkram Rajabhat University
156 Moo 5, Tambon Phlai Chumphon, Amphur Mueang, Phitsanulok 65000, Thailand
natthinee@psru.ac.th

[2]Department of Mathematics
Faculty of Sciences and Agricultural Technology
Rajamangala University of Technology Lanna
52 Moo 7, Tambon Ban Krang, Amphur Mueang, Phitsanulok 65000, Thailand
*Corresponding author: pk_g@rmutl.ac.th

Received April 2023; accepted July 2023

Abstract. *The focus of this paper is to present the idea of a neutrosophic bipolar-valued fuzzy subsemigroup. We examine the concepts of the neutrosophic bipolar-valued fuzzy left (right, interior) ideal and demonstrate that they align with regular and intra-regular semigroups. Additionally, we introduce the notion of a neutrosophic bipolar-valued fuzzy simple and establish that a semigroup is only considered simple if it is neutrosophic bipolar-valued fuzzy simple.*

Keywords: Neutrosophic sets, Bipolar fuzzy ideal, Subsemigroups, Regular semigroup, Intra-regular semigroup

1. **Introduction.** In the realm of mathematics, sets are used to represent collections of objects that share common properties. However, in real-world scenarios, the properties of objects are often uncertain or vague. To handle such uncertainties, the concept of fuzzy sets was introduced by Zadeh [1] in 1965. Fuzzy sets allow for the representation of degrees of membership of an object to a set rather than just a binary classification of whether the object belongs to the set or not. Building upon the concept of fuzzy sets, Atanassov [2] introduced intuitionistic fuzzy sets, which generalize fuzzy sets by considering not only the degree of membership but also the degree of non-membership of an object to a set. Neutrosophic sets, introduced by Smarandache [3], further extend the concept of fuzzy sets by representing truth-membership, indeterminacy-membership, and falsity-membership of an object to a set independently. These concepts have been applied to various algebraic structures, including fields, rings, vector spaces, groups, and semigroups [4, 5, 6, 7, 8, 9, 10, 11]. In particular, fuzzy sets in semigroups were introduced and studied by Kuroki [12], who investigated fuzzy (left, right) ideals and fuzzy bi-ideals in semigroups.

In this context, the concept of a bipolar fuzzy semigroup by Zhang [13], which allows for the representation of degrees of membership, degrees of non-membership, and degrees of partial membership simultaneously, is a helpful extension of classical, fuzzy, and neutrosophic semigroups. Moreover, it has potential applications in handling uncertainties and partial knowledge in various fields. Recently, in 2021, Gaketem and Khamrot [14] introduced the concepts of bipolar fuzzy weakly interior ideals of semigroups. The relationship between bipolar fuzzy weakly interior ideals and bipolar fuzzy left (right) ideals

DOI: 10.24507/icicel.18.04.343

and the relationship between bipolar fuzzy weakly interior ideals and bipolar fuzzy interior ideals are also discussed. Furthermore, Gaketem et al. [15] introduced the concept of bipolar fuzzy implicative UP-filters in UP-algebras. Based on these notions, bipolar fuzzy set theory and its applications were developed [16, 17, 18].

This paper introduces neutrosophic bipolar-valued fuzzy sets in semigroups. Firstly, definitions of neutrosophic bipolar-valued fuzzy ideals and neutrosophic bipolar-valued fuzzy interior ideals of semigroups are provided. Then we prove that the neutrosophic bipolar-valued fuzzy ideals and the neutrosophic bipolar-valued fuzzy interior ideals coincide with regular and intra-regular semigroups. Lastly, we introduce the concept of a neutrosophic bipolar-valued fuzzy simple in the semigroup. We characterize and prove a simple semigroup in terms of the neutrosophic bipolar-valued fuzzy interior ideal.

2. **Problem Statement and Preliminaries.** In this section, we give definitions that are used in this paper. By a subsemigroup of a semigroup S, we mean a non-empty subset A of S such that $A^2 \subseteq A$, and by a left (right) ideal of S, we mean a non-empty subset A of S such that $SA \subseteq A$ ($AS \subseteq A$). By a two-sided ideal or simply an ideal, we mean a non-empty subset of a semigroup S that is both a left and a right ideal of S. A non-empty subset A of S is called an interior ideal of S if $SAS \subseteq A$. A semigroup S is called regular if, for all $a \in S$, there exists $x \in S$ such that $a = axa$. A semigroup S is called intra-regular if, for all $a \in S$, there exists $x, y \in S$ such that $a = xa^2y$.

Zadeh studied the theory of fuzzy sets in 1965 [1], in which he defined as follows: A fuzzy set ω of a non-empty set F is a function from F into the closed interval $[0,1]$, i.e., $\omega : F \to [0,1]$.

Definition 2.1. *A* bipolar fuzzy set *(shortly, BF set) ω on X is an object having the form*

$$\omega := \left\{ \left(x, \omega^+(x), \omega^-(x)\right) \middle| \, x \in X \right\},$$

where $\omega^+ : X \to [0,1]$ and $\omega^+ : X \to [-1,0]$.

Definition 2.2. [3] *Let X be a non-empty set. A neutrosophic sets (NS) A in X is the structure*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T_A : X \to [0,1]$ is a truth membership function, $I_A : X \to [0,1]$ is an indeterminate membership function, and $F_A : X \to [0,1]$ is a false membership function.

3. **Neutrosophic Bipolar-Valued Fuzzy Sets in Semigroups.** In this section, we shall introduce the fundamental operations that can be carried out on neutrosophic bipolar-valued fuzzy sets of the semigroup. For brevity, we will employ the abbreviated term NBF instead of repeatedly using the full term "neutrosophic bipolar-valued fuzzy set".

Definition 3.1. [3] *Let X be a non-empty set. A neutrosophic bipolar-valued fuzzy set (NBF) A in X is an object of the form*

$$A = \left\{ \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) \right\rangle : x \in X \right\},$$

where $T_A^+, I_A^+, F_A^+ : X \to [0,1]$ and $T_A^-, I_A^-, F_A^- : X \to [-1,0]$.

For simplicity, we use the symbol $A = (A^+, A^-)$ for the NBF

$$A = \left\{ \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) \right\rangle : x \in X \right\}.$$

Definition 3.2. *An NBF set $A = (A^+, A^-)$ in a semigroup S is called an NBF subsemigroup if it satisfies*

$$(\forall x, y \in S) \begin{pmatrix} T_A^+(xy) \geq T_A^+(x) \wedge T_A^+(y), \\ I_A^+(xy) \leq I_A^+(x) \vee I_A^+(y), \\ F_A^+(xy) \geq F_A^+(x) \wedge F_A^+(y), \\ T_A^-(xy) \leq T_A^-(x) \vee T_A^-(y), \\ I_A^-(xy) \geq I_A^-(x) \wedge I_A^-(y), \\ F_A^-(xy) \leq F_A^-(x) \vee F_A^-(y) \end{pmatrix}$$

Example 3.1. *Consider a semigroup $S = \{z_1, z_2, z_3\}$ with the following Cayley table:*

$\star$	z_1	z_2	z_3
z_1	z_3	z_3	z_3
z_2	z_3	z_3	z_1
z_3	z_3	z_2	z_3

Define an NBF $A = (A^+, A^-)$ in S as follows:

S	T_A^+	I_A^+	F_A^+	T_A^-	I_A^-	F_A^-
z_1	0.3	0.5	0.6	-0.4	-0.6	-0.8
z_2	0.2	0.3	0.8	-0.6	-0.7	-0.6
z_3	0.7	0.8	0.5	-0.2	-0.3	-0.9

Then $A = (A^+, A^-)$ is an NBF subsemigroup of S.

Definition 3.3. *Let NBF $A = (A^+, A^-)$ in a semigroup S and $\mu_1, \mu_2, \mu_3 \in [0, 1]$, $\delta_1, \delta_2, \delta_3 \in [-1, 0]$, the sets*

$$\left(T_A^+\right)^{\mu_1} = \left\{k \in S \middle| T_A^+(k) \geq \mu_1\right\},$$
$$\left(I_A^+\right)^{\mu_2} = \left\{k \in S \middle| I_A^+(k) \leq \mu_2\right\},$$
$$\left(F_A^+\right)^{\mu_3} = \left\{k \in S \middle| F_A^+(k) \geq \mu_3\right\}.$$

The set

$$P_A^+(\mu_1, \mu_2, \mu_3) := \left\{k \in S \middle| T_A^+(k) \geq \mu_1, I_A^+(k) \leq \mu_2, F_A^+(k) \geq \mu_3\right\}$$

is called a positive (μ_1, μ_2, μ_3)-level of $A = (A^+, A^-)$. It is evident that $P_A^+(\mu_1, \mu_2, \mu_3) = \left(T_A^+\right)^{\mu_1} \cap \left(I_A^+\right)^{\mu_2} \cap \left(F_A^+\right)^{\mu_3}$, and

$$\left(T_A^-\right)^{\delta_1} = \left\{k \in S \middle| T_A^-(k) \leq \delta_1\right\},$$
$$\left(I_A^-\right)^{\delta_2} = \left\{k \in S \middle| I_A^-(k) \geq \delta_2\right\},$$
$$\left(F_A^-\right)^{\delta_3} = \left\{k \in S \middle| F_A^-(k) \leq \delta_3\right\}.$$

The set

$$N_A^-(\delta_1, \delta_2, \delta_3) := \left\{k \in S \middle| T_A^-(k) \leq \delta_1, I_A^-(k) \geq \delta_2, F_A^-(k) \leq \delta_3\right\}$$

is called a negative $(\delta_1, \delta_2, \delta_3)$-level of $A = (A^+, A^-)$. It is evident that $N_A^-(\delta_1, \delta_2, \delta_3) = \left(T_A^-\right)^{\delta_1} \cap \left(I_A^-\right)^{\delta_2} \cap \left(F_A^-\right)^{\delta_3}$.

The set

$$C_A^{\pm}(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3) = P_A^+(\mu_1, \mu_2, \mu_3) \cap N_A^-(\delta_1, \delta_2, \delta_3)$$

is called the bipolar $(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3)$-level of $A = (A^+, A^-)$.

Definition 3.4. *An NBF set $A = (A^+, A^-)$ in a semigroup S is called an NBF right (left) ideal if it satisfies*

$$(\forall x, y \in S) \begin{pmatrix} T_A^+(xy) \geq T_A^+(x)(T_A^+(xy) \geq T_A^+(y)), \\ I_A^+(xy) \leq I_A^+(x)(I_A^+(xy) \leq I_A^+(y)), \\ F_A^+(xy) \geq F_A^+(x)(F_A^+(xy) \geq F_A^+(y)), \\ T_A^-(xy) \leq T_A^-(x)(T_A^-(xy) \leq T_A^-(y)), \\ I_A^-(xy) \geq I_A^-(x)(I_A^-(xy) \geq I_A^-(y)), \\ F_A^-(xy) \leq F_A^-(x)(F_A^-(xy) \leq F_A^-(y)) \end{pmatrix}$$

By a (two-sided) NBF ideal, we mean a left and right NBF ideal.

Example 3.2. *Consider a semigroup $S = \{z_1, z_2, z_3\}$ with the following Cayley table:*

$\star$	z_1	z_2	z_3
z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1
z_3	z_1	z_1	z_3

Define an NBF $A = (A^+, A^-)$ in S as follows:

S	T_A^+	I_A^+	F_A^+	T_A^-	I_A^-	F_A^-
z_1	0.7	0.8	0.1	−0.2	−0.3	−0.9
z_2	0.2	0.3	0.2	−0.6	−0.7	−0.7
z_3	0.1	0.5	0.2	−0.7	−0.5	−0.8

It is easy to verify that $A = (A^+, A^-)$ is an NBF ideal of S. Every right (resp. left) NBF ideal is an NBF subsemigroup. However, the converse may not be true, as seen in the following example.

Example 3.3. *Consider a semigroup $S = \{z_1, z_2, z_3, z_4\}$ with the following Cayley table:*

$\star$	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1	z_1
z_3	z_1	z_1	z_1	z_2
z_4	z_1	z_1	z_2	z_3

Define an NBF $A = (A^+, A^-)$ in S as follows:

S	T_A^+	I_A^+	F_A^+	T_A^-	I_A^-	F_A^-
z_1	0.5	0.7	0.1	−0.2	−0.1	−0.3
z_2	0.3	0.4	0.3	−0.6	−0.7	−0.4
z_3	0.5	0.5	0.2	−0.4	−0.5	−0.4
z_4	0.2	0.2	0.5	−0.6	−0.8	−0.5

It is easy to verify that $A = (A^+, A^-)$ is an NBF subsemigroup of S, but it is not a left NBF ideal of S, since $T_A^+(z_4 z_3) = T_A^+(z_2) = 0.3 < 0.5 = T_A^+(z_3)$.

Definition 3.5. *Let S be a semigroup. An NBF subsemigroup $A = (A^+, A^-)$ in S is an NBF interior ideal in S if the assertions below are valid:*

$$(\forall a, x, y \in S) \begin{pmatrix} T_A^+(xay) \geq T_A^+(a), \\ I_A^+(xay) \leq I_A^+(a), \\ F_A^+(xay) \geq F_A^+(a), \\ T_A^-(xay) \leq T_A^-(a), \\ I_A^-(xay) \geq I_A^-(a), \\ F_A^-(xay) \leq F_A^-(a) \end{pmatrix}$$

Theorem 3.1. *Every NBF ideal of a semigroup S is an NBF interior ideal of S.*

Definition 3.6. *For any non-empty subset K of set X, the characteristic NBF function of K in X is defined to be a structure*

$$\chi_K = \left\{ \left\langle x, T^+_{\chi_K}(x), I^+_{\chi_K}(x), F^+_{\chi_K}(x), T^-_{\chi_K}(x), I^-_{\chi_K}(x), F^-_{\chi_K}(x) \right\rangle : x \in X \right\},$$

where

$$T^+_{\chi_K} : X \to [0,1]; x \mapsto T^+_{\chi_K}(x) := \begin{cases} 1 & \text{if } x \in K \\ 0 & \text{if } x \notin K, \end{cases}$$

$$I^+_{\chi_K} : X \to [0,1]; x \mapsto I^+_{\chi_K}(x) := \begin{cases} 0 & \text{if } x \in K \\ 1 & \text{if } x \notin K, \end{cases}$$

$$F^+_{\chi_K} : X \to [0,1]; x \mapsto F^+_{\chi_K}(x) := \begin{cases} 1 & \text{if } x \in K \\ 0 & \text{if } x \notin K, \end{cases}$$

$$T^-_{\chi_K} : X \to [-1,0]; x \mapsto T^-_{\chi_K}(x) := \begin{cases} -1 & \text{if } x \in K \\ 0 & \text{if } x \notin K, \end{cases}$$

$$I^-_{\chi_K} : X \to [-1,0]; x \mapsto I^-_{\chi_K}(x) := \begin{cases} 0 & \text{if } x \in K \\ -1 & \text{if } x \notin K, \end{cases}$$

$$F^-_{\chi_K} : X \to [-1,0]; x \mapsto F^-_{\chi_K}(x) := \begin{cases} -1 & \text{if } x \in K \\ 0 & \text{if } x \notin K. \end{cases}$$

For simplicity, we use the symbol $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ for the characteristic NBF (shortly, CNBF) function $\chi_K = \left\{ \left\langle x, T^+_{\chi_K}(x), I^+_{\chi_K}(x), F^+_{\chi_K}(x), T^-_{\chi_K}(x), I^-_{\chi_K}(x), F^-_{\chi_K}(x) \right\rangle : x \in X \right\}$. The semigroup S can be considered a fuzzy subset of itself, i.e., $\chi_{\mathcal{S}}(x) = \langle 1, 1, 0, -1, -1, 0 \rangle$ for all $x \in S$.

Definition 3.7. *Let $A = (A^+, A^-)$ and $B = (B^+, B^-)$ be an NBF in a semigroup S, Then*

1. *$A = (A^+, A^-)$ is called an NBF in $B = (B^+, B^-)$, denoted by $A \sqsubseteq B = (A^+ \sqsubseteq B^+, A^- \sqsubseteq B^-)$ if $T^+_A(x) \le T^+_B(x)$, $I^+_A(x) \ge I^+_B(x)$, $F^+_A(x) \le F^+_B(x)$, $T^-_A(x) \ge T^-_B(x)$, $I^-_A(x) \le I^-_B(x)$, $F^-_A(x) \ge F^-_B(x)$, for all $x \in S$. If $A \sqsubseteq B$ and $B \sqsubseteq A$, then we say that $A = B$.*
2. *The union of two NBF $A = (A^+, A^-)$ and $B = (B^+, B^-)$ is defined as*
   $A \sqcup B = (A^+ \sqcup B^+, A^- \sqcup B^-) = \left\{ \left\langle x, \left(T^+_A \cup T^+_B\right)(x), \left(I^+_A \cup I^+_B\right)(x), \left(F^+_A \cup F^+_B\right)(x), \left(T^-_A \cup T^-_B\right)(x), \left(I^-_A \cup I^-_B\right)(x), \left(F^-_A \cup F^-_B\right)(x) \right\rangle : x \in X \right\}$, *where* $\forall x \in S$,
   $\left(T^+_A \cup T^+_B\right)(x) = T^+_A(x) \vee T^+_B(x)$, $\left(I^+_A \cup I^+_B\right)(x) = I^+_A(x) \wedge I^+_B(x)$, $\left(F^+_A \cup F^+_B\right)(x) = F^+_A(x) \vee F^+_B(x)$,
   $\left(T^-_A \cup T^-_B\right)(x) = T^-_A(x) \wedge T^-_B(x)$, $\left(I^-_A \cup I^-_B\right)(x) = I^-_A(x) \vee I^-_B(x)$, $\left(F^-_A \cup F^-_B\right)(x) = F^-_A(x) \wedge F^-_B(x)$.
3. *The intersection of two NBF $A = (A^+, A^-)$ and $B = (B^+, B^-)$ is defined as*
   $A \sqcap B = (A^+ \sqcap B^+, A^- \sqcap B^-) = \left\{ \left\langle x, \left(T^+_A \cap T^+_B\right)(x), \left(I^+_A \cap I^+_B\right)(x), \left(F^+_A \cap F^+_B\right)(x), \left(T^-_A \cap T^-_B\right)(x), \left(I^-_A \cap I^-_B\right)(x), \left(F^-_A \cap F^-_B\right)(x) \right\rangle : x \in X \right\}$, *where* $\forall x \in S$,
   $\left(T^+_A \cap T^+_B\right)(x) = T^+_A(x) \wedge T^+_B(x)$, $\left(I^+_A \cap I^+_B\right)(x) = I^+_A(x) \vee I^+_B(x)$, $\left(F^+_A \cap F^+_B\right)(x) = F^+_A(x) \wedge F^+_B(x)$,
   $\left(T^-_A \cap T^-_B\right)(x) = T^-_A(x) \vee T^-_B(x)$, $\left(I^-_A \cap I^-_B\right)(x) = I^-_A(x) \wedge I^-_B(x)$, $\left(F^-_A \cap F^-_B\right)(x) = F^-_A(x) \vee F^-_B(x)$.

Lemma 3.1. *If K is a subsemigroup of a semigroup S, then the CNBF function $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF subsemigroup of S.*

Proof: Suppose that K is a subsemigroup of S and let $x, y \in S$.

If $x, y \in K$, then $xy \in K$. Thus, $1 = T^+_{\chi_K}(x) = T^+_{\chi_K}(y) = T^+_{\chi_K}(xy)$, $0 = I^+_{\chi_K}(x) = I^+_{\chi_K}(y) = I^+_{\chi_K}(xy)$, $1 = F^+_{\chi_K}(x) = F^+_{\chi_K}(y) = F^+_{\chi_K}(xy)$, $-1 = T^-_{\chi_K}(x) = T^-_{\chi_K}(y) = T^-_{\chi_K}(xy)$, $0 = I^-_{\chi_K}(x) = I^-_{\chi_K}(y) = I^-_{\chi_K}(xy)$, and $-1 = F^-_{\chi_K}(x) = F^-_{\chi_K}(y) = F^-_{\chi_K}(xy)$.

Hence, $T^+_K(xy) \geq T^+_K(x) \wedge T^+_K(y)$, $I^+_K(xy) \leq I^+_K(x) \vee I^+_K(y)$, $F^+_K(xy) \geq F^+_K(x) \wedge F^+_K(y)$, $T^-_K(xy) \leq T^-_K(x) \vee T^-_K(y)$, $I^-_K(xy) \geq I^-_K(x) \wedge I^-_K(y)$, $F^-_K(xy) \leq F^-_K(x) \vee F^-_K(y)$.

If $x \notin K$ or $y \notin K$, then $T^+_K(xy) \geq T^+_K(x) \wedge T^+_K(y)$, $I^+_K(xy) \leq I^+_K(x) \vee I^+_K(y)$, $F^+_K(xy) \geq F^+_K(x) \wedge F^+_K(y)$, $T^-_K(xy) \leq T^-_K(x) \vee T^-_K(y)$, $I^-_K(xy) \geq I^-_K(x) \wedge I^-_K(y)$, $F^-_K(xy) \leq F^-_K(x) \vee F^-_K(y)$. Thus, $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF subsemigroup of S. □

Lemma 3.2. *If $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF subsemigroup of S, then K is a subsemigroup in a semigroup S.*

Proof: Suppose that $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF subsemigroup of S, and let $x, y \in S$. Then $T^+_{\chi_K}(x) = T^+_{\chi_K}(y) = 1$, $I^+_{\chi_K}(x) = I^+_{\chi_K}(y) = 1$, $F^+_{\chi_K}(x) = F^+_{\chi_K}(y) = 0$, $T^-_{\chi_K}(x) = T^-_{\chi_K}(y) = -1$, $I^-_{\chi_K}(x) = I^-_{\chi_K}(y) = -1$, and $F^-_{\chi_K}(x) = F^-_{\chi_K}(y) = 0$. By assumption,

$$\begin{cases} T^+_K(xy) \geq T^+_K(x) \wedge T^+_K(y), I^+_K(xy) \leq I^+_K(x) \vee I^+_K(y), F^+_K(xy) \geq F^+_K(x) \wedge F^+_K(y), \\ T^-_K(xy) \leq T^-_K(x) \vee T^-_K(y), I^-_K(xy) \geq I^-_K(x) \wedge I^-_K(y), F^-_K(xy) \leq F^-_K(x) \vee F^-_K(y). \end{cases} \tag{1}$$

If $xy \notin K$, then by (1), $0 = T^+_K(xy) \geq 1$, $1 = I^+_K(xy) \leq 0$, $0 = F^+_K(xy) \geq 1$, $0 = T^-_K(xy) \leq -1$, $-1 = I^-_K(xy) \geq 0$, $0 = F^-_K(xy) \leq -1$. It is a contradiction. Hence, $xy \in K$. Therefore, K is a subsemigroup in S. □

The following result is an immediate consequence of Lemma 3.1 and Lemma 3.2.

Theorem 3.2. *Let K be a non-empty subset of a semigroup S. Then K is a subsemigroup of S if and only if $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF subsemigroup of S.*

Theorem 3.3. *Let S be a semigroup. Then, for any $K \subseteq S$, the given assertions are equivalent:*

1. *K is a right ideal (resp., left ideal),*
2. *$\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF right ideal (resp., NBF left ideal).*

Proof: (1⇒2) Suppose that K is a right ideal of S and $x, y \in S$. If $x \in K$, then $T^+_{\chi_K}(xy) \geq T^+_{\chi_K}(x) = 1$, $I^+_{\chi_K}(xy) \leq I^+_{\chi_K}(x) = 0$, $F^+_{\chi_K}(xy) \geq F^+_{\chi_K}(x) = 1$, $T^-_{\chi_K}(xy) \leq T^-_{\chi_K}(x) = -1$, $I^-_{\chi_K}(xy) \geq I^-_{\chi_K}(x) = 0$, and $F^-_{\chi_K}(xy) \leq F^-_{\chi_K}(x) = -1$. If $x \notin K$, then $T^+_{\chi_K}(xy) \geq T^+_{\chi_K}(x) = 0$, $I^+_{\chi_K}(xy) \leq I^+_{\chi_K}(x) = 1$, $F^+_{\chi_K}(xy) \geq F^+_{\chi_K}(x) = 0$, $T^-_{\chi_K}(xy) \leq T^-_{\chi_K}(x) = 0$, $I^-_{\chi_K}(xy) \geq I^-_{\chi_K}(x) = -1$, and $F^-_{\chi_K}(xy) \leq F^-_{\chi_K}(x) = 0$. By Definition 3.6, $\chi_K = \left(\chi^+_A, \chi^-_K\right)$ is an NBF right ideal.

(2⇒1) Assume $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF right ideal. Let $x \in K$ and $y \in S$. Then $T^+_{\chi_K}(x) = 1$, $I^+_{\chi_K}(x) = 0$, $F^+_{\chi_K}(x) = 1$, $T^-_{\chi_K}(x) = -1$, $I^-_{\chi_K}(x) = 0$, and $F^-_{\chi_K}(x) = -1$, which imply $xy \in K$. Hence, by Definition 3.6, K is a right ideal. □

Theorem 3.4. *Let S be a semigroup. Then for any $K \subseteq S$, the given assertions are equivalent:*

1. *K is an interior ideal,*
2. *$\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF interior ideal.*

Proof: (1⇒2) Suppose that K is an interior ideal of S, let $x, y, a \in S$. If $a \in K$, then $T^+_{\chi_K}(xay) \geq T^+_{\chi_K}(a) = 1$, $I^+_{\chi_K}(xay) \leq I^+_{\chi_K}(a) = 0$, $F^+_{\chi_K}(xay) \geq F^+_{\chi_K}(a) = 1$, $T^-_{\chi_K}(xay) \leq T^-_{\chi_K}(a) = -1$, $I^-_{\chi_K}(xay) \geq I^-_{\chi_K}(a) = 0$, and $F^-_{\chi_K}(xay) \leq F^-_{\chi_K}(a) = -1$. If $a \notin K$, then $T^+_{\chi_K}(xay) \geq T^+_{\chi_K}(a) = 0$, $I^+_{\chi_K}(xay) \leq I^+_{\chi_K}(a) = 1$, $F^+_{\chi_K}(xay) \geq F^+_{\chi_K}(a) = 0$, $T^-_{\chi_K}(xay) \leq T^-_{\chi_K}(a) = 0$, $I^-_{\chi_K}(xay) \geq I^-_{\chi_K}(a) = -1$, and $F^-_{\chi_K}(xay) \leq F^-_{\chi_K}(a) = 0$. By Definition 3.6, $\chi_K = \left(\chi^+_K, \chi^-_K\right)$ is an NBF interior ideal.

(2⇒1) Assume $\chi_K = \left(\chi_K^+, \chi_K^-\right)$ is an NBF interior ideal. Let $x, y, a \in S$. Then $T^+_{\chi_K}(a) = 1$, $I^+_{\chi_K}(a) = 0$, $F^+_{\chi_K}(a) = 1$, $T^-_{\chi_K}(a) = -1$, $I^-_{\chi_K}(a) = 0$, and $F^-_{\chi_K}(a) = -1$, which imply $xay \in K$. Hence, by Definition 3.6, K is an interior ideal. □

Theorem 3.5. *Let S be a semigroup. Then the arbitrary intersection (resp., union) of NBF interior ideals in S is an NBF interior ideal of S.*

Proof: The proof is a routine procedure. □

Theorem 3.6. *Let S be a semigroup. If S is regular, then NBF interior ideals of S are NBF ideals.*

Proof: Assume $A = (A^+, A^-)$ is an NBF interior ideal of S, and let $x, y \in S$. As $y \in S$ and S is regular, there is $r \in S$ such that $y = yry$.

Now, $T^+_A(xy) \geq T^+_A(xyry) \geq T^+_A(y)$, $I^+_A(xy) \leq I^+_A(xyry) \leq I^+_A(y)$, $F^+_A(xy) \geq F^+_A(xyry) \geq F^+_A(y)$, $T^-_A(xy) \leq T^-_A(xyry) \leq T^-_A(y)$, $I^-_A(xy) \geq I^-_A(xyry) \geq I^-_A(y)$, $F^-_A(xy) \leq F^-_A(xyry) \leq F^-_A(y)$. Therefore, $A = (A^+, A^-)$ is an NBF left ideal. In a similar way, we can claim that $A = (A^+, A^-)$ is an NBF right ideal. Hence, $A = (A^+, A^-)$ is an NBF ideal of S. □

Theorem 3.7. *Let S be a semigroup. If S is an intra-regular, then NBF interior ideals of S are NBF ideals.*

Proof: Assume $A = (A^+, A^-)$ is an NBF interior ideal of S, and let $x, y \in S$. As $y \in S$ and S is an intra-regular, there exist $s, t \in S$ such that $y = sy^2t$.

Now, $T^+_A(xy) \geq T^+_A(xsy^2t) \geq T^+_A(y)$, $I^+_A(xy) \leq I^+_A(xsy^2t) \leq I^+_A(y)$, $F^+_A(xy) \geq F^+_A(xsy^2t) \geq F^+_A(y)$, $T^-_A(xy) \leq T^-_A(xsy^2t) \leq T^-_A(y)$, $I^-_A(xy) \geq I^-_A(xsy^2t) \geq I^-_A(y)$, $F^-_A(xy) \leq F^-_A(xsy^2t) \leq F^-_A(y)$. Therefore, $A = (A^+, A^-)$ is an NBF left ideal. Similarly, we can claim that $A = (A^+, A^-)$ is an NBF right ideal. Hence, $A = (A^+, A^-)$ is an NBF ideal of S. □

Definition 3.8. *A semigroup S is said to be*

1. *left (resp., right) simple if it does not contain any proper left (resp., right) ideal of S.*
2. *simple if it does not contain any proper ideal of S.*

Definition 3.9. *A semigroup S is known as an NBF simple if all the NBF ideals are constant functions, i.e., for any NBF ideal $A = (A^+, A^-)$ in S, we can have $T^+_A(x) = T^+_A(y)$, $I^+_A(x) = I^+_A(y)$, $F^+_A(x) = F^+_A(y)$, $T^-_A(x) = T^-_A(y)$, $I^-_A(x) = I^-_A(y)$ and $F^-_A(x) = F^-_A(y)$ for all $x, y \in S$.*

Let S be a semigroup. Then, for any $k \in S$, we define $J_k \subseteq S$ as follows:

$J_k := \left\{m \in S \middle| T^+_A(m) \geq T^+_A(k), I^+_A(m) \leq I^+_A(k), F^+_A(m) \geq F^+_A(k), T^-_A(m) \leq T^-_A(k), I^-_A(m) \geq I^-_A(k), F^-_A(m) \leq F^-_A(k)\right\}$.

Theorem 3.8. *Let S be a semigroup. If $A = (A^+, A^-)$ is an NBF right ideal (resp., left ideal, ideal) of S, then, for any J_k is a right ideal (resp., left ideal, ideal) in S.*

Proof: Let $k \in S$, then clearly $\emptyset \neq J_k \subseteq S$. Let $x \in J_k$ and $y \in S$, then $xy \in J_k$. Since $x, y \in S$ and $A = (A^+, A^-)$ is an NBF right ideal, we get $T^+_A(xy) \geq T^+_A(x)$, $I^+_A(xy) \leq I^+_A(x)$, $F^+_A(xy) \geq F^+_A(x)$, $T^-_A(xy) \leq T^-_A(x)$, $I^-_A(xy) \geq I^-_A(x)$, $F^-_A(xy) \leq F^-_A(x)$. Since $x \in J_k$, we get $T^+_A(x) \geq T^+_A(k)$, $I^+_A(x) \leq I^+_A(k)$, $F^+_A(x) \geq F^+_A(k)$, $T^-_A(x) \leq T^-_A(k)$, $I^-_A(x) \geq I^-_A(k)$, $F^-_A(x) \leq F^-_A(k)$ which imply that $xy \in J_k$. Therefore, J_k is the right ideal in S. □

Theorem 3.9. *If S is a semigroup, then $S = (S^+, S^-)$ is an NBF simple if and only if S is simple.*

Proof: Suppose that $S = (S^+, S^-)$ is an NBF simple. Let J be ideal in S. Then, by Theorem 3.3, $\chi_S = \left(\chi_S^+, \chi_S^-\right)$ is an NBF ideal. We now prove that $S = J$. Let $k \in S$. Since S is an NBF simple, $\chi_S = \left(\chi_S^+, \chi_S^-\right)$ is constant and $\chi_S(k) = \chi_S(k')$ for every $k' \in S$. In particular, we have $T^+_{\chi_A}(k) = T^+_{\chi_A}(d) = 1$, $I^+_{\chi_A}(k) = I^+_{\chi_A}(d) = 0$, $F^+_{\chi_A}(k) = F^+_{\chi_A}(d) = 1$, $T^-_{\chi_A}(k) = T^-_{\chi_A}(d) = -1$, $I^-_{\chi_A}(k) = I^-_{\chi_A}(d) = 0$, and $F^-_{\chi_A}(k) = F^-_{\chi_A}(d) = -1$ for any $d \in J$ which gives $k \in J$. Thus, $S \subseteq J$, and hence $S = J$.

Conversely, let $S = (S^+, S^-)$ is an NBF ideal with $x, y \in S$. Then, by Theorem 3.8, J_x is ideal. As S is simple, we have $J_x = S$. Since $y \in J_x$, we have $T^+_A(y) \geq T^+_A(x)$, $I^+_A(y) \leq I^+_A(x)$, $F^+_A(y) \geq F^+_A(x)$, $T^-_A(y) \leq T^-_A(x)$, $I^-_A(y) \geq I^-_A(x)$, $F^-_A(y) \leq F^-_A(x)$. So, $T^+_A(y) = T^+_A(x)$, $I^+_A(y) = I^+_A(x)$, $F^+_A(y) = F^+_A(x)$, $T^-_A(y) = T^-_A(x)$, $I^-_A(y) = I^-_A(x)$, $F^-_A(y) = F^-_A(x)$. Hence, $S = (S^+, S^-)$ is an NBF simple. □

Lemma 3.3. *A semigroup S is simple if and only if $S = SaS$ for all $a \in S$.*

Theorem 3.10. *For any semigroup S, S is simple if and only if all the NBF interior ideals of S are constant functions.*

Proof: Suppose $x, y \in S$ and S are simple. Let $A = (A^+, A^-)$ be an NBF ideal. Then, by Lemma 3.3, we get $S = SxS = SyS$. Since $x \in SxS$, we get $x = tys$ for $t, s \in S$. Since $A = (A^+, A^-)$ is an NBF interior ideal, we have $T^+_A(x) \geq T^+_A(tys) \geq T^+_A(y)$, $I^+_A(x) \leq I^+_A(tys) \leq I^+_A(y)$, $F^+_A(x) \geq F^+_A(tys) \geq F^+_A(y)$, $T^-_A(x) \leq T^-_A(tys) \leq T^-_A(y)$, $I^-_A(x) \geq I^-_A(tys) \geq I^-_A(y)$, $F^-_A(x) \leq F^-_A(tys) \leq F^-_A(y)$. Similarly, we can prove that $T^+_A(y) \geq T^+_A(x)$, $I^+_A(y) \leq I^+_A(x)$, $F^+_A(y) \geq F^+_A(x)$, $T^-_A(y) \leq T^-_A(x)$, $I^-_A(y) \geq I^-_A(x)$, $F^-_A(y) \leq F^-_A(x)$. So, $A = (A^+, A^-)$ is constant.

Conversely, suppose $A = (A^+, A^-)$ is an NBF ideal of S. Then $A = (A^+, A^-)$ is an NBF interior ideal. By assumption, $A = (A^+, A^-)$ is constant, and hence $A = (A^+, A^-)$ is an NBF simple. Therefore, S is simple, by Theorem 3.9. □

As a consequence, we have the following.

Theorem 3.11. *For a semigroup S, the following is equivalent:*

1. *S is simple.*
2. *$S = SaS$ for every $a \in S$.*
3. *S is an NBF simple.*
4. *For every NBF interior ideal of S, $T^+_A(xay) \geq T^+_A(a)$, $I^+_A(xay) \leq I^+_A(a)$, $F^+_A(xay) \geq F^+_A(a)$, $T^-_A(xay) \leq T^-_A(a)$, $I^-_A(xay) \geq I^-_A(a)$, $F^-_A(xay) \leq F^-_A(a)$ for all $a, x, y \in S$.*

Theorem 3.12. *Let S be a semigroup. If $A = (A^+, A^-)$ is an NBF interior ideal with $\mu_1, \mu_2, \mu_3 \in [0, 1]$ and $\delta_1, \delta_2, \delta_3 \in [-1, 0]$, $0 \leq \mu_1 + \mu_2 + \mu_3 \leq 3$ and $-3 \leq \delta_1 + \delta_2 + \delta_3 \leq 0$, then $(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3)$-level set in $A = (A^+, A^-)$ is an NBF interior ideal provided $C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3) \neq \emptyset$.*

Proof: Suppose $C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3) \neq \emptyset$ for $\mu_1, \mu_2, \mu_3 \in [0, 1]$ and $\delta_1, \delta_2, \delta_3 \in [-1, 0]$.

Let $A = (A^+, A^-)$ be an NBF interior ideal of S and $x, y \in S$, $a \in C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3)$. Then $T^+_A(xay) \geq T^+_A(a) \geq \mu_1, I^+_A(xay) \leq I^+_A(a) \leq \mu_2, F^+_A(xay) \geq F^+_A(a) \geq \mu_3$, $T^-_A(xay) \leq T^-_A(a) \leq \delta_1, I^-_A(xay) \geq I^-_A(a) \geq \delta_2$, and $F^-_A(xay) \leq F^-_A(a) \leq \delta_3$ which imply $xay \in C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3)$.

Therefore, $C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3)$ is an NBF interior ideal in S. □

Theorem 3.13. *Let S be a semigroup. $C^{\pm}_A(\mu_1, \mu_2, \mu_3, \delta_1, \delta_2, \delta_3) \neq \emptyset$ and $A = (A^+, A^-)$ is an NBF of S with $\mu_1, \mu_2, \mu_3 \in [0, 1]$ and $\delta_1, \delta_2, \delta_3 \in [-1, 0]$ such that $0 \leq \mu_1 + \mu_2 + \mu_3 \leq 3$ and $-3 \leq \delta_1 + \delta_2 + \delta_3 \leq 0$. If $\left(T^+_A\right)^{\mu_1}$, $\left(I^+_A\right)^{\mu_2}$, $\left(F^+_A\right)^{\mu_3}$, $\left(T^-_A\right)^{\delta_1}$, $\left(I^-_A\right)^{\delta_2}$, $\left(F^-_A\right)^{\delta_3}$ are interior ideals on S, then $A = (A^+, A^-)$ is an NBF interior ideal on S whenever $\left(T^+_A\right)^{\mu_1} \neq \emptyset$, $\left(I^+_A\right)^{\mu_2} \neq \emptyset$, $\left(F^+_A\right)^{\mu_3} \neq \emptyset$, $\left(T^-_A\right)^{\delta_1} \neq \emptyset$, $\left(I^-_A\right)^{\delta_2} \neq \emptyset$, $\left(F^-_A\right)^{\delta_3} \neq \emptyset$.*

Proof: Suppose that, for $x, y, a \in S$ with $\left(T_A^+\right)^{\mu_1}(xay) \leq \left(T_A^+\right)^{\mu_1}(a)$. Then $\left(T_A^+\right)^{\mu_1}(xay) \leq \mu_1 \leq \left(T_A^+\right)^{\mu_1}(a)$ for some $\mu_1 \in [0,1]$. So, $a \in \left(T_A^+\right)^{\mu_1}$, but $xay \notin \left(T_A^+\right)^{\mu_1}$ is a contradiction. Thus, $\left(T_A^+\right)^{\mu_1}(xay) \geq \left(T_A^+\right)^{\mu_1}(a)$.

Suppose that, for $x, y, a \in S$ with $\left(I_A^+\right)^{\mu_2}(xay) \geq \left(I_A^+\right)^{\mu_2}(a)$. Then $\left(I_A^+\right)^{\mu_2}(xay) \geq \mu_2 \geq \left(I_A^+\right)^{\mu_2}(a)$ for some $\mu_2 \in [0,1]$. So, $a \in \left(I_A^+\right)^{\mu_2}$, but $xay \notin \left(I_A^+\right)^{\mu_2}$ is a contradiction. Thus, $\left(I_A^+\right)^{\mu_2}(xay) \leq \left(I_A^+\right)^{\mu_2}(a)$.

Suppose that, for $x, y, a \in S$ with $\left(F_A^+\right)^{\mu_3}(xay) \leq \left(F_A^+\right)^{\mu_3}(a)$. Then $\left(F_A^+\right)^{\mu_3}(xay) \leq \mu_3 \leq \left(F_A^+\right)^{\mu_3}(a)$ for some $\mu_3 \in [0,1]$. So, $a \in \left(F_A^+\right)^{\mu_3}$, but $xay \notin \left(F_A^+\right)^{\mu_3}$ is a contradiction. Thus, $\left(F_A^+\right)^{\mu_3}(xay) \geq \left(F_A^+\right)^{\mu_3}(a)$.

Suppose that, for $x, y, a \in S$ with $\left(T_A^-\right)^{\delta_1}(xay) \geq \left(T_A^-\right)^{\delta_1}(a)$. Then $\left(T_A^-\right)^{\delta_1}(xay) \geq \delta_1 \geq \left(T_A^-\right)^{\delta_1}(a)$ for some $\delta_1 \in [-1,0]$. So, $a \in \left(T_A^-\right)^{\delta_1}$, but $xay \notin \left(T_A^-\right)^{\delta_1}$ is a contradiction. Thus, $\left(T_A^-\right)^{\delta_1}(xay) \leq \left(T_A^-\right)^{\delta_1}(a)$.

Suppose that, for $x, y, a \in S$ with $\left(I_A^-\right)^{\delta_2}(xay) \leq \left(I_A^-\right)^{\delta_2}(a)$. Then $\left(I_A^-\right)^{\delta_2}(xay) \leq \delta_2 \leq \left(I_A^-\right)^{\delta_2}(a)$ for some $\delta_2 \in [-1,0]$. So, $a \in \left(I_A^-\right)^{\delta_2}$, but $xay \notin \left(I_A^-\right)^{\delta_2}$ is a contradiction. Thus, $\left(I_A^-\right)^{\delta_2}(xay) \geq \left(I_A^-\right)^{\delta_2}(a)$.

Suppose that, for $x, y, a \in S$ with $\left(F_A^-\right)^{\delta_3}(xay) \geq \left(F_A^-\right)^{\delta_3}(a)$. Then $\left(F_A^-\right)^{\delta_3}(xay) \geq \delta_3 \geq \left(F_A^-\right)^{\delta_3}(a)$ for some $\delta_3 \in [-1,0]$. So, $a \in \left(F_A^-\right)^{\delta_3}$, but $xay \notin \left(F_A^-\right)^{\delta_3}$ is a contradiction. Thus, $\left(F_A^-\right)^{\delta_3}(xay) \leq \left(F_A^-\right)^{\delta_3}(a)$.

Hence, $A = (A^+, A^-)$ is an NBF interior ideal of S. □

4. **Conclusion.** This paper has presented the concept of a neutrosophic bipolar-valued fuzzy subsemigroup and its basic operations. The concepts of the neutrosophic bipolar-valued fuzzy left (right, interior) ideal has been discussed and shown to coincide with regular and intra-regular semigroups. Furthermore, the idea of a neutrosophic bipolar-valued fuzzy simple has been introduced. It has been proved that a semigroup is considered simple if and only if it is neutrosophic bipolar-valued fuzzy simple. Further, we extend to fuzzy bi-interior ideals, fuzzy almost ideals, and algebraic systems. The study of neutrosophic bipolar-valued fuzzy sets in semigroup theory opens up a new area of research and paves the way for further investigation in this field.

Acknowledgment. This work is partially supported by Pibulsongkram Rajabhat University, Fundamental Fund Year 2023 Capital Code RDI-1-66-21. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, *Inf. Control.*, vol.8, pp.338-353, 1965.
[2] K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, vol.20, pp.87-96, 1986.
[3] F. Smarandache, A unifying field in logics: Neutrosophic logic, in *Philosophy*, American Research Press, 1999.
[4] B. Elavarasan, F. Smarandache and Y. B. Jun, Neutrosophic ℵ-ideals in semigroups, *Neutrosophic Sets Syst.*, vol.28, pp.273-280, 2019.
[5] W. V. Kandasamy and K. Ilanthenral, *Smarandashe Special Elements in Multiset Semigroups*, EuropaNova ASBL, Brussels, Belgium, 2018.
[6] W. B. V. Kandasamy, I. Kandasamy and F. Smarandache, Semi-idempotents in neutrosophic rings, *Mathematics*, vol.7, p.507, 2019.
[7] W. B. V. Kandasamy, I. Kandasamy and F. Smarandache, Neutrosophic triplets in neutrosophic rings, *Mathematics*, vol.7, p.563, 2019.
[8] W. B. V. Kandasamy, I. Kandasamy and F. Smarandache, Neutrosophic quadruple vector spaces and their properties, *Mathematics*, vol.7, p.758, 2019.

[9] S. H. Nasseri, I. Mahdavi, Z. A. Afrouzy and R. Nourifar, A fuzzy mathematical multi-period multi-echelon supply chain model based on extension principle, *Ann. Univ. Craiova Math.*, vol.42, no.2, pp.384-401, 2015.
[10] F. Smarandache and M. Ali, Neutrosophic triplet group, *Neural. Comput. Appl.*, vol.29, no.7, pp.595-601, 2018.
[11] N. Tiprachot, S. Lekkoksung, N. Lekkoksung and B. Pibaljommee, Regularities in terms of hybrid n-interior ideals and hybrid (m, n)-ideals of ordered semigroups, *International Journal of Innovative Computing, Information and Control*, vol.18, no.5, pp.1347-1362, DOI: 10.24507/ijicic.18.05.1347, 2022.
[12] N. Kuroki, Fuzzy bi-ideals in semigroup, *Comment. Math. Univ. St. Pauli.*, vol.5, pp.128-132, 1979.
[13] W. R. Zhang, Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis, *Proc. of the 1st International Joint Conference of the North American Fuzzy Information Processing Society Biannual Conference (NAFIPS/IFIS/NASA'94)*, pp.305-309, 1994.
[14] T. Gaketem and P. Khamrot, On some semigroups characterized in terms of bipolar fuzzy weakly interior ideals, *IAENG Int. J. Appl. Math.*, vol.48, no.2, pp.250-256, 2021.
[15] T. Gaketem, P. Khamrot, P. Julatha and A. Iampan, Bipolar fuzzy comparative up-filters, *IAENG Int. J. Appl. Math.*, vol.52, no.3, pp.1-6, 2022.
[16] N. Deetae, Multiple criteria decision making based on bipolar fuzzy sets application to fuzzy TOPSIS, *Int. J. Math. Comput. Sci.*, vol.12, p.29, 2021.
[17] P. Khamrot and M. Siripitukdet, On properties of generalized bipolar fuzzy semigroups, *Warasan Songkhla Nakharin*, vol.41, no.2, pp.405-413, 2019.
[18] P. Khamrot and M. Siripitukdet, Some types of subsemigroups characterized in terms of inequalities of generalized bipolar fuzzy subsemigroups, *Mathematics*, vol.5, p.71, 2017.