ICIC Express Letters ICIC International ©2024 ISSN 1881-803X
Volume **18**, Number **5**, May **2024** pp. **503–511**

FAULT DETECTION OF NONLINEAR SYSTEMS BASED ON TIME-VARYING THRESHOLDS AND NEURAL NETWORK OBSERVER

Meie Wang, Zhihui Zhang* and Jing Xie

College of Artificial Intelligence
Shenyang University of Technology
No. 111, Shenliao West Road, Economic & Technological Development Zone
Shenyang 110870, P. R. China
{ wangmeie0410; 1984xiejing }@163.com
*Corresponding author: zhihuizhang1988@163.com

Received August 2023; accepted October 2023

Abstract. *This paper advanced a fault detection (FD) strategy based on neural network (NN) observer and a time-varying threshold for unknown nonlinear systems. Firstly, NN is applied to approximating the unknown nonlinear function and FD observer is constructed with recursive algorithm. Secondly, the time-varying thresholds are calculated with the application of the prescribed performance bound (PPB) and the FD decision is proposed. Fault detectability is analyzed simultaneously. The proposed time-varying threshold method reduces false alarms induced by overshoot within transients and alerts as early as possible compared to the method that does without considering PPB. Finally, the availability of the approach is demonstrated by the comparison result of the simulation instances.*
Keywords: Fault detection, Nonlinear system, Time varying threshold, Neural network observer

1. **Introduction.** Nonlinear systems exist in various practical control systems, and how to solve the FD problem in nonlinear systems has become one of the popular study topics among scholars. In long-term research and accumulation, most of the research results express the model-based FD method [1], whose central ideas are to produce the residual signal, calculate the threshold, and diagnose whether the fault occurs by the residual evaluation function. However, in the existence of unknown nonlinearities, modeling the system becomes challenging. Within this field, neural networks (NNs) can efficiently approximate complex nonlinear systems due to the basis function mapping relationship in the hidden layer, which has shown to be extremely advantageous in FD [2-4]. For example, the NN-based observer is constructed to solve actuator FD in an unknown input affine nonlinear system [5]. In [6,7], based on deep NN, the fault diagnosis method is proposed to improve the diagnosis accuracy and control distribution efficiency. Based on the optimal interval model, an adaptive FD method is proposed to address issues with traditional FD methods in nonlinear dynamic systems with parameter uncertainties [8]. In [9], a recurrent NN-based fault diagnosis method is proposed for control systems of actuator fault diagnosis. The above research has made great achievements in the study of nonlinear systems by exploiting the ability of NNs to approximate nonlinear functions.

The selection of thresholds is an important research topic for FD systems. In [10], an adaptive threshold based on statistical methods is achieved by linearizing a nonlinear model. A time varying threshold method has been proposed by the analytical solution of a nonlinear system in [11]. In [12,13], the random time-varying detection thresholds

DOI: 10.24507/icicel.18.05.503

are designed for nonlinear systems to reduce the level of false alarm probability. In recent years, a PPB-based controller design approach is proposed in [14,15]. This approach ensures that the PPB of tracking error is always satisfied during operation. In [16], the PPB-based observer is presented for uncertain nonlinear systems and the time-varying threshold-based FD approach is proposed with prescribed performance. To sum up, it is worth studying how to design time-varying thresholds for unknown nonlinear systems using prescribed performance functions.

In this paper, an FD approach based on NN observer and time-varying threshold is presented for nonlinear systems with unmeasurable states, unknown functions and mismatched fault functions. The basic contributions of this paper are as below. 1) The NN weight update laws and observer gain functions are designed by using recursive algorithms. 2) Considering the prescribed performance of residual signals, a time-varying threshold is presented based on PPB. 3) The presented time-varying threshold-based approach enhances the FD capability and reduces false alarms resulting from overshoot during transients than that without considering PPB.

The structure of this paper is as below. Section 2 gives the system and the structure of NN. Section 3 designs residual generator based on NN and provides the FD scheme by calculating the time-varying threshold based on PPB. Section 4 provides the simulation examples, and Section 5 presents the conclusions.

2. **Problem Statement.** Regard the following problem of fault detection for unknown nonlinear systems:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) + \omega_1(t) + \phi_1(t-T)g_1\left(\bar{x}_1(t)\right) \\
\dot{x}_2(t) &= x_3(t) + \omega_2(t) + \phi_2(t-T)g_2\left(\bar{x}_2(t)\right) \\
&\vdots \\
\dot{x}_{n-1}(t) &= x_n(t) + \omega_{n-1}(t) + \phi_{n-1}(t-T)g_{n-1}\left(\bar{x}_{n-1}(t)\right) \\
\dot{x}_n(t) &= a(x) + u(t) + \omega_n(t) + \phi_n(t-T)g_n\left(x(t)\right) \\
y(t) &= x_1(t),
\end{aligned} \tag{1}$$

where $x(t) = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the system state, $u(t) \in R$ is the input of the system. $\omega_i(t)$ $(i = 1, 2, \ldots, n)$ are the disturbance uncertainties. $a(x)$ is unknown nonlinear function. Let $\bar{x}_i(t) = [x_1, x_2, \ldots, x_i]^T \in R^i$, $i = 1, 2, \ldots, n$, and $\phi_i(t-T)g_i(\bar{x}_i(t))$, $i = 1, 2, \ldots, n$ describes the fault function. The following assumptions are given for the system (1).

Assumption 2.1. *The unknown interferences $\omega_i(t)$ meet $|\omega_i(t)| \le G_i$, $i = 1, 2, \ldots, n$ with G_i being known positive constants.*

Considering the unknown nonlinear function within the system, the NN is applied to approximating $a(x)$ as $a(x) = W^{*T}S(x) + \delta$, where $S(x) = [S_j(x)]^T$ is the Gaussian basis function output, $S_j(x) = \exp\left(\frac{-\|x-\mu_j\|^2}{2\nu_j^2}\right)$, W^* is the ideal weights, and δ is the bounded NN approximation error.

Assumption 2.2. *The existence of normal numbers W_1 and S_1 makes the ideal weights W^* and the activation function $S(x)$ are bounded, that is, $\|W^*\| \le W_1$ and $\|S(x)\| \le S_1$.*

Introduce the prescribed performance function described in [14, 15], which is selected as $\varphi(t) = (\varphi_0 - \varphi_\infty) e^{-\lambda t} + \varphi_\infty$, where φ_0, φ_∞ and λ are the selectable positive constants. An error conversion method is presented to convert the residual signal, which is limited by the prescribed performance, into an equivalent unconstrained signal. Define

$$e_y(t) = \varphi(t)P(\xi), \tag{2}$$

where $\xi(t)$ is the converted residual and $P(\xi)$ is a rigorously increasing function. Moreover, the inverse conversion is $\xi(t) = P^{-1}\left(e_y(t)/\varphi(t)\right)$. The differential conversion of the residual $e_y(t)$ is shown as below:

$$\dot{\xi}(t) = \dot{e}_y(t) - \dot{\varphi}(t)P(\xi)/\varphi(t)(\partial P/\partial\xi). \tag{3}$$

3. Main Results.

3.1. Neural network residual generator.

Based on the NN observer, the structure of the residual generator is designed as below:

$$\begin{aligned} \dot{\hat{x}}_1(t) &= \hat{x}_2(t) + N_n\Psi(\cdot) \\ \dot{\hat{x}}_2(t) &= \hat{x}_3(t) + N_{n-1}\Psi(\cdot) \\ &\vdots \\ \dot{\hat{x}}_{n-1}(t) &= \hat{x}_n(t) + N_2\Psi(\cdot) \\ \dot{\hat{x}}_n(t) &= \hat{a}(\hat{x}) + u(t) + N_1\Psi(\cdot) \\ \hat{y}(t) &= \hat{x}_1(t), \end{aligned} \tag{4}$$

where $N_1, N_2, \ldots, N_n$ and $\Psi(\cdot)$ are the observer gain and nonlinear function to be designed, respectively. $\hat{a}(\hat{x})$ is an online approximation model with adjustable weights as below:

$$\hat{a}(\hat{x}) = \hat{W}^T(t)S(\hat{x}). \tag{5}$$

Defining $e_i(t) = \hat{x}_i(t) - x_i(t)$, $i = 1, 2, \ldots, n$ and $\tilde{W} = \hat{W} - W^*$, the error dynamics are obtained as below:

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) + N_n\Psi(\cdot) - \omega_1(t) \\ \dot{e}_2(t) &= e_3(t) + N_{n-1}\Psi(\cdot) - \omega_2(t) \\ &\vdots \\ \dot{e}_{n-1}(t) &= e_n(t) + N_2\Psi(\cdot) - \omega_{n-1}(t) \\ \dot{e}_n(t) &= \Delta a(x) + N_1\Psi(\cdot) - \omega_n(t) \\ e_y(t) &= \hat{y}(t) - y(t), \end{aligned} \tag{6}$$

where $\Delta a(x) = \hat{a}(\hat{x}) - a(x) = \tilde{W}^T(t)S(\hat{x}) + \eta - \delta$, $\eta = W^T\tilde{S}$, η is bounded.

Considering the conversion (2) and (3), the new residual dynamics is given as below:

$$\begin{aligned} \dot{\xi}(t) &= \frac{1}{\varphi(t)(\partial P/\partial\xi)}\left[\dot{e}_y(t) - \dot{\varphi}(t)P(\xi)\right] \\ &= \frac{1}{\varphi(t)(\partial P/\partial\xi)}\left[e_2(t) + N_n\Psi(\cdot) - \dot{\varphi}(t)P(\xi) - \omega_1(t)\right]. \end{aligned} \tag{7}$$

The following assumptions are required before proceeding with the observer design.

Assumption 3.1. *There exist a constant N_1 and positive constants ν_1, ℓ_1, θ, a C^1 function $V_1\left(\tilde{W}_n(t), e_n(t)\right)$ which is quadratic in $e_n(t)$, satisfying*

$$\begin{aligned} & D_{\tilde{W}_n}V_1 \cdot \dot{\tilde{W}}_n(t) + D_{e_n}V_1 \cdot \dot{e}_n(t) \\ &= D_{\tilde{W}_n}V_1 \cdot \left[-\left(e_n(t)S_n\left(\hat{\bar{x}}_n(t)\right) + \hat{W}_n\right)\right] + D_{e_n}V_1 \cdot \left[\tilde{W}_n^T(t)S_n\left(\hat{\bar{x}}_n(t)\right) + \eta - \delta - N_1e_n(t)\right] \\ &\le -\nu_1|e_n(t)|^2 - \ell_1\left|\tilde{W}_n(t)\right|^2 + \theta. \end{aligned} \tag{8}$$

The theorem below presents the gain function of the observer, the NN weight update law, and stability analysis results.

Theorem 3.1. *Regard the system (1) satisfying Assumptions 2.1, 2.2 and 3.1, the NN weight update law is designed*

$$\dot{\hat{W}}_1 = -\left(e_1 S_1(\hat{x}) + \hat{W}_1\right), \tag{9}$$

the residual generator (4) with $N_1 = q_1 q_2 \cdots q_{n-1}, N_2 = q_2 \cdots q_{n-1}, \ldots, N_{n-1} = q_{n-1}, N_n = 1$ *and*

$$\Psi(\cdot) = -\varphi(t)(\partial P/\partial\xi)\xi(t)q_n - \dot{\varphi}(t)P(\xi) - \hat{W}_1(t)S_1(\hat{x}(t)). \tag{10}$$

If there exist positive constants $\zeta_1, \zeta_2, \ldots, \zeta_{4n+2}$, $z_1, z_2, \ldots, z_{n-1}$ *satisfying* $q_2, \ldots, q_{i+1}$ $(3 \leq i \leq n-2)$ *and* q_n, *then* $e_n(t), \ldots, e_2(t)$, $\xi(t)$ *and* $\tilde{W}_1, \tilde{W}_2, \ldots, \tilde{W}_n$ *are uniformly bounded in the absence of faults, and* e_y *is within the predetermined PPB.*

Proof: Step 1. Let $e_1(t) = e_2(t) = \cdots = e_{n-2}(t) = 0$, $N_2 = 1$, and the last two equations of (6) are

$$\begin{aligned} \dot{e}_{n-1}(t) &= e_n(t) + \Psi(\cdot) - \omega_{n-1}(t) \\ \dot{e}_n(t) &= \Delta a(x) + N_1\Psi(\cdot) - \omega_n(t). \end{aligned} \tag{11}$$

Using coordinate conversion $\begin{bmatrix} \varsigma_{n-1}(t) \\ \varsigma_n(t) \end{bmatrix} = K_1 \begin{bmatrix} e_{n-1}(t) \\ e_n(t) \end{bmatrix}$, where $K_1 = \begin{bmatrix} 1 & 0 \\ -N_1 & 1 \end{bmatrix}$, then Equation (11) is converted as below:

$$\begin{aligned} \dot{\zeta}_{n-1}(t) &= \varsigma_n(t) + N_1\varsigma_{n-1}(t) + \Psi(\cdot) - \omega_{n-1}(t)\dot{\zeta}_n(t) \\ &= \Delta a(x) - N_1^2\varsigma_{n-1}(t) - N_1\varsigma_n(t) + N_1\omega_{n-1}(t) - \omega_n(t). \end{aligned} \tag{12}$$

Constructing a virtual NN weight update law as $\dot{\hat{W}}_{n-1} = -\left(\varsigma_{n-1}(t)S_{n-1}\left(\hat{\bar{x}}_{n-1}\right) + \hat{W}_{n-1}\right)$ and choosing the Lyapunov function

$$\bar{V}_2\left(\tilde{W}_n(t), \tilde{W}_{n-1}(t), \varsigma_n(t), \varsigma_{n-1}(t)\right) = \bar{V}_1\left(\tilde{W}_n(t), \varsigma_n(t)\right) + \frac{1}{2}\varsigma_{n-1}^T(t)\varsigma_{n-1}(t) + \frac{1}{2}\tilde{W}_{n-1}^T\tilde{W}_{n-1},$$

the derivative of $\bar{V}_2$ is as below:

$$\begin{aligned} \dot{\bar{V}}_2 \leq{}& -\tilde{\nu}_2|\zeta_n|^2 - \tilde{\ell}_2\left|\tilde{W}_n\right|^2 + \left(\frac{1}{\zeta_1} + \frac{\zeta_2}{4}\left(\mu_2 N_1^2 + 1\right)^2 + \frac{1}{\zeta_3} + \frac{1}{\zeta_5} + \frac{1}{\zeta_6} + N_1\right)|\varsigma_{n-1}|^2 \\ &- \left(\frac{1}{2} - \frac{\zeta_5}{4}|S_{n-1}|^2 - \frac{\zeta_6}{4}|S_{n-1}|^2\right)\left|\tilde{W}_{n-1}\right|^2 + \bar{G}_2(G_n, G_{n-1}) + \zeta_{n-1}^T\Psi(\cdot) \\ &+ \zeta_{n-1}^T\hat{W}_{n-1}S_{n-1}\left(\hat{\bar{x}}_{n-1}\right), \end{aligned} \tag{13}$$

where $\tilde{\nu}_2 = \nu_1 - \frac{1}{\zeta_2} - \frac{1}{\zeta_4} > 0$, $\tilde{\ell}_2 = \ell_1 - \frac{\zeta_1}{4}\mu_1^2 N_1^2|S_n|^2 > 0$ and

$$\bar{G}_2(G_n, G_{n-1}) = \frac{\zeta_4}{4}\mu_2^2\left(N_1 G_{n-1} + G_n\right)^2 + \frac{\zeta_3}{4}G_{n-1}^2 + \theta + \frac{1}{2}\|W_{n-1}\|^2 + \frac{\zeta_6}{4}\|W_{n-1}\|^2|S_{n-1}|^2.$$

Let $\frac{1}{\zeta_1} + \frac{\zeta_2}{4}\left(\mu_2 N_1^2 + 1\right)^2 + \frac{1}{\zeta_3} + \frac{1}{\zeta_5} + \frac{1}{\zeta_6} + N_1 + z_1 = q_2$ and $\Psi(\cdot) = -q_2\zeta_{n-1} - \hat{W}_{n-1}S_{n-1}\left(\hat{\bar{x}}_{n-1}\right)$, $\dot{\bar{V}}_2$ can be written as

$$\begin{aligned} \dot{\bar{V}}_2 \leq{}& -\tilde{\nu}_2|\varsigma_n|^2 - z_1|\zeta_{n-1}|^2 - \tilde{\ell}_2\left|\tilde{W}_n\right|^2 - \left(\frac{1}{2} - \frac{\zeta_5}{4}|S_{n-1}|^2 - \frac{\zeta_6}{4}|S_{n-1}|^2\right)\left|\tilde{W}_{n-1}\right|^2 \\ &+ \bar{G}_2(G_n, G_{n-1}) \\ \leq{}& -\bar{\nu}_2\|(\varsigma_n, \zeta_{n-1})\|^2 - \bar{\ell}_2\left\|\left(\tilde{W}_n, \tilde{W}_{n-1}\right)\right\|^2 + \bar{G}_2\left(G_n, G_{n-1}\right). \end{aligned} \tag{14}$$

Step i $(3 \leq i \leq n-2)$. Selecting the NN weight update law as $\dot{\hat{W}}_{n-i} = -\Big(e_{n-i}S_{n-i}\left(\hat{\bar{x}}_{n-i}\right) + \hat{W}_{n-i}\Big)$, choosing the Lyapunov function $\bar{V}_{i+1} = \bar{V}_i + \frac{1}{2}\varsigma_{n-i}^T(t)\varsigma_{n-i}(t) + \frac{1}{2}\tilde{W}_{n-i}^T\tilde{W}_{n-i}$, designing the observer gain $\frac{\zeta_{4i-1}}{4}+\frac{1}{\zeta_{4i}}+\frac{1}{\zeta_{4i+1}}+\frac{1}{\zeta_{4i+2}}+N_1+z_i=q_{i+1}$ and $\Psi(*)=-q_{i+1}e_{n-i}(t)-\hat{W}_{n-i}(t)S_{n-i}\left(\hat{\hat{x}}_{n-i}(t)\right)$, $\dot{\bar{V}}_{i+1}$ can rewrite

$$\dot{\bar{V}}_{i+1}\le-\nu_{i+1}\|(e_n,\ldots,e_{n-i})\|^2-\bar{\beta}_{i+1}\left\|\left(\tilde{W}_n,\ldots,\tilde{W}_{n-i}\right)\right\|^2+\bar{G}_{i+1}(G_n,\ldots,G_{n-i}), \tag{15}$$

where $\bar{\nu}_i-\frac{1}{\zeta_{4i-1}}>0$, $\bar{\nu}_{i+1}=\min\left\{\bar{\nu}_i,\bar{\nu}_i-\frac{1}{\zeta_{4i-1}},\kappa_i\right\}$, $\nu_{i+1}=\bar{\nu}_{i+1}\left\|K_i\right\|^2$ and

$$\bar{\ell}_{i+1}=\min\left\{\bar{\ell}_i,\frac{1}{2}-\frac{\zeta_{4i+1}}{4}\left|S_{n-i}\right|^2-\frac{\zeta_{4i+2}}{4}\left|S_{n-i}\right|^2\right\}.$$

Step $n-1$. Let $N_1=q_1q_2\cdots q_{n-1},N_2=q_2\cdots q_{n-1},\ldots,N_n=1$. In this step, the recursive algorithm obtains implementable NN weight update laws (10). According to the coordinate conversion, it is clear that $\xi_1(t)=\varsigma_1(t)$.

Constructing the Lyapunov function

$$\bar{V}_n\left(\tilde{W}(t),\varsigma(t)\right)=\bar{V}_{n-1}\left(\tilde{W}_n(t),\ldots,\tilde{W}_2(t),\varsigma_n(t),\ldots,\varsigma_2(t)\right)+\frac{1}{2}\varsigma_1^T(t)\varsigma_1(t)+\frac{1}{2}\tilde{W}_1^T\tilde{W}_1,$$

the derivative of $\bar{V}_n$ can be obtained and let

$$\begin{aligned}&\frac{1}{\zeta_{4n}}\frac{1}{\varphi^2(\partial P/\partial\xi)^2}+\frac{\zeta_{4n-1}}{4}\frac{1}{\varphi^2(\partial P/\partial\xi)^2}+\frac{1}{\zeta_{4n+1}}\frac{1}{\varphi^2(\partial P/\partial\xi)^2}+\frac{1}{\zeta_{4n+2}}\frac{1}{\varphi^2(\partial P/\partial\xi)^2}\\&+\frac{N_1}{\varphi(\partial P/\partial\xi)}+z_{n-1}=q_n.\end{aligned} \tag{16}$$

By designing $\Psi(\cdot)$ and q_n satisfying Equations (10) and (16), respectively, $\dot{\bar{V}}_n$ can be written as

$$\begin{aligned}\dot{\bar{V}}_n\le&-\bar{\nu}_{n-1}\left(\left|\varsigma_n\right|^2+\cdots+\left|\varsigma_3\right|^2\right)-\hat{\nu}_{n-1}\left|\varsigma_2\right|^2-z_{n-1}\left|\varsigma_1\right|^2-\bar{\ell}_{n-1}\left(\left|\tilde{W}_n\right|^2+\cdots+\left|\tilde{W}_3\right|^2\right)\\&-\hat{\ell}_{n-1}\left|\tilde{W}_2\right|^2-\sigma\left\|\tilde{W}_1\right\|^2+\bar{G}_n(G_n,\ldots,G_1),\end{aligned} \tag{17}$$

where $\hat{\ell}_{n-1}=\bar{\ell}_{n-1}-\frac{1}{2}+\frac{\zeta_{4n-3}+\zeta_{4n-2}}{4}\left|S_2\right|^2>0$, $\sigma=\frac{1}{2}-\frac{\zeta_{4n+1}}{4}\left|S_1\right|^2-\frac{\zeta_{4n+2}}{4}\left|S_1\right|^2$, $\hat{\nu}_{n-1}=\bar{\nu}_{n-1}-\frac{\zeta_{4n-1}}{4}>0$, $\bar{G}_n(G_n,\ldots,G_1)=\bar{G}_{n-1}(G_n,\ldots,G_2)+\frac{\zeta_{4n}}{4}G_1^2+\frac{1}{2}\left\|W_1\right\|^2+\frac{\zeta_{4n+2}}{4}\left\|W_1\right\|^2\left|S_1\right|^2$.

It follows immediately from Equation (17) that $\dot{\bar{V}}_n$ is negative outside the tight set of $\Sigma_{(\varsigma_1,\varsigma_2,\ldots,\varsigma_n)}$ and $\Sigma_{\left(\tilde{W}_1,\tilde{W}_2,\ldots,\tilde{W}_n\right)}$:

$$\begin{aligned}&\Sigma_{(\varsigma_1,\varsigma_2,\ldots,\varsigma_n)}\\&=\left\{(\xi,\varsigma_2,\ldots,\varsigma_n)\left||\xi|\le\sqrt{\frac{\bar{G}_n}{z_{n-1}}},|\varsigma_2|\le\sqrt{\frac{\bar{G}_n}{\hat{\nu}_{n-1}}},\|(\varsigma_3,\ldots,\varsigma_n)\|\le\sqrt{\frac{\bar{G}_n}{\bar{\nu}_{n-1}}}\right.\right\},\end{aligned} \tag{18}$$

$$\begin{aligned}&\Sigma_{\left(\tilde{W}_1,\tilde{W}_2,\ldots,\tilde{W}_n\right)}\\&=\left\{\left(\tilde{W}_1,\tilde{W}_2,\ldots,\tilde{W}_n\right)\left|\left|\tilde{W}_1\right|\le\sqrt{\frac{\bar{G}_n}{\sigma}},\left|\tilde{W}_2\right|\le\sqrt{\frac{\bar{G}_n}{\hat{\ell}_{n-1}}},\left\|\left(\tilde{W}_3,\ldots,\tilde{W}_n\right)\right\|\le\sqrt{\frac{\bar{G}_n}{\bar{\ell}_{n-1}}}\right.\right\},\end{aligned} \tag{19}$$

that is, $(\xi,\varsigma_2,\ldots,\varsigma_n)$ and $\left(\tilde{W}_1,\tilde{W}_2,\ldots,\tilde{W}_n\right)$ are uniformly bounded. According to the coordinate conversion, it follows that $(\xi,e_2,\ldots,e_n)$ is also uniformly bounded and $e_y(t)$ is within the pre-given PPB.

Corollary 3.1. *Regard Equation (1) satisfying Assumptions 2.1, 2.2 and 3.1, the NN weight update law is designed as* $\dot{\hat{W}}_1=-\left(e_1S_1\left(\hat{\hat{x}}_1\right)+\hat{W}_1\right)$*, the residual generator (4) with* $N_1=q_1q_2\cdots q_{n-1},N_2=q_2\cdots q_{n-1},\ldots,N_n=1$ *and*

$$\Psi'(*)=-q_n'\varsigma_1(t)-\hat{W}_1(t)S_1(\hat{x}(t)) \tag{20}$$

If there exist positive constants $\zeta_1, \zeta_2, \dots, \zeta_{4n+2}$*,* $z_1, z_2, \dots, z_{n-1}$*, then* $e_n(t), \dots, e_2(t), e_y(t)$ *are uniformly bounded in the absence of faults and the residual signal satisfies* $|e_y(t)| \le \sqrt{\frac{\bar{G}_n}{z_{n-1}}}$.

Proof: Depending on Equation (6) and applying the identical design steps from Step 1 to Step $n-2$ in Theorem 3.1, the difference is Step $n-1$: Let $N_1 = q_1 q_2 \cdots q_{n-1}, N_2 = q_2 \cdots q_{n-1}, \dots, N_n = 1$, all equations of (6) can be obtained. It is apparent that $e_1(t) = \varsigma_1(t)$. Selecting the Lyapunov function $\bar{V}_n\left(\tilde{W}(t), \varsigma(t)\right) = \bar{V}_{n-1}\left(\tilde{W}_n(t), \dots, \tilde{W}_2(t), \varsigma_n(t), \dots, \varsigma_2(t)\right) + \frac{1}{2}\varsigma_1^T(t)\varsigma_1(t) + \frac{1}{2}\tilde{W}_1^T\tilde{W}_1$, the derivative of $\bar{V}_n$ along the solution of Equation (6) can be obtained. Let

$$\frac{\zeta_{4n-1}}{4} + \frac{1}{\zeta_{4n}} + \frac{1}{\zeta_{4n+1}} + \frac{1}{\zeta_{4n+2}} + N_1 + z_{n-1} = q_n', \tag{21}$$

constructing $\Psi'(\cdot)$ and q_n' satisfying Equations (20) and (21), respectively, $\dot{\bar{V}}_n$ is given as below:

$$\begin{aligned} \dot{\bar{V}}_n \le & -\bar{\nu}_{n-1}\left(|\varsigma_n|^2 + \cdots + |\varsigma_3|^2\right) - \hat{\nu}_{n-1}|\varsigma_2|^2 - z_{n-1}|\varsigma_1|^2 - \bar{\ell}_{n-1}\left(\left\|\tilde{W}_n\right\|^2 + \cdots + \left\|\tilde{W}_3\right\|^2\right) \\ & - \hat{\ell}_{n-1}\left\|\tilde{W}_2\right\|^2 - \sigma\left\|\tilde{W}_1\right\|^2 + \bar{G}_n\left(G_n, \dots, G_1\right), \end{aligned} \tag{22}$$

which means that $\dot{\bar{V}}_n$ is negative beyond the following compact set (19) and (23):

$$\begin{aligned} & \Sigma_{(\varsigma_1, \varsigma_2, \dots, \varsigma_n)} \\ & = \left\{ (\varsigma_1, \varsigma_2, \dots, \varsigma_n) \middle| |\varsigma_1| \le \sqrt{\frac{\bar{G}_n}{z_{n-1}}}, |\varsigma_2| \le \sqrt{\frac{\bar{G}_n}{\hat{\nu}_{n-1}}}, \|(\varsigma_3, \dots, \varsigma_n)\| \le \sqrt{\frac{\bar{G}_n}{\bar{\nu}_{n-1}}} \right\}. \end{aligned} \tag{23}$$

This proof is completed.

3.2. **Fault detection analysis.** In this section, the FD strategy is given the NN-based observer in Theorem 3.1. Retrospecting the error conversion $e_y(t) = \varphi(t)P(\xi)$ and defining $\bar{\xi} = \sqrt{\frac{\bar{G}_n}{z_{n-1}}}$, it is true that $|P(\xi)| \le P\left(\bar{\xi}\right)$; therefore, $|e_y(t)| \le \varphi(t)P\left(\bar{\xi}\right)$.

The fault detection decision logic: The fault alarm is generated when $e_y(t)$ is beyond of the following threshold range: $\varphi(t)P\left(\bar{\xi}\right) < e_y(t) < -\varphi(t)P\left(\bar{\xi}\right)$.

Similar to [17], the following theorem provides detectability conditions, based on the above analysis.

Theorem 3.2. *If there exists* $[T + t_1, T + t_2]$ $(t_2 > t_1 > 0)$ *such that the fault satisfies*

$$\begin{aligned} & \left\| \int_{T+t_1}^{T+t_2} Ce^{A(T+t_2-\tau)}\phi(t-T)g(x(\tau))d\tau \right\| \\ & \ge \left(1 + ke^{-\gamma(t_2-t_1)}\right)\varphi(t)P\left(\bar{\xi}\right) + ke^{-\gamma(t_2-\hbar_1)}\sum_{i=1}^{n}\varsigma_i + \frac{k\vartheta}{\gamma}\left[1 - e^{-\gamma(t_2-t_1)}\right] \\ & \quad + (t_2 - t_1)\sum_{i=1}^{n} G_i, \end{aligned} \tag{24}$$

then the fault can be monitored, that is $|e_y(T + t_2)| > \varphi(t)P\left(\bar{\xi}\right)$.

Proof: Write Equation (6) in the compact form:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + \Delta a\left(\tilde{W}(t), e(t)\right) - \phi(t-T)g(x(t)) - \omega(t) \\ r(t) &= Ce(t). \end{aligned} \tag{25}$$

The solution of Equation (25) is

$$\begin{aligned} e_y(T+t_2) &= Ce^{A(t_2-t_1)}e(T+t_1) \\ &\quad + \int_{T+t_1}^{T+t_2} Ce^{A(T+t_2-\tau)}\left[\Delta a\left(\tilde{W}(\tau),x(\tau)\right)-\omega(\tau)-\phi(t-T)g(x(\tau))\right]d\tau. \end{aligned}$$

From Theorem 3.1, we have $|e_y(T+t_1)| \le \varphi(t)P(\bar{\xi})$ and $|e_i(T+t_1)| \le \epsilon_i$ $(i=2,\ldots,n)$, where ϵ_i denote the bounds of the estimation errors, and then $\left\|\Delta a\left(\tilde{W}(t),x(t)\right)\right\| \le \vartheta$ holds. Thus, it is clear that

$$\begin{aligned} |e_y(T+t_2)| &\ge -ke^{-\gamma(t_2-t_1)}\left(\varphi(t)P\left(\bar{\xi}\right)+\sum_{i=2}^{n}\epsilon_i\right)-\frac{k\vartheta}{\gamma}\left[1-e^{-\gamma(t_2-t_1)}\right]-(t_2-t_1)\sum_{i=1}^{n}G_i \\ &\quad + \left\|\int_{T+t_1}^{T+t_2} Ce^{A(T+t_2-\tau)}\phi(t-T)g(x(\tau))d\tau\right\|, \end{aligned}$$

where $k > 0$ and $\gamma > 0$, so that $\left\|Ce^{At}\right\| \le ke^{-\gamma t}$. If the faults meet Equation (25), $|e_y(T+t_2)| > \varphi(t)P\left(\bar{\xi}\right)$ can be implemented directly.

4. **Simulation Results.** Consider the unknown nonlinear systems below:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t)+0.5\sin(26t)+20.5\cos\left(100x_1^2(t)\right)\phi_1(t-T) \\ \dot{x}_2(t) &= x_3(t)+0.1\cos(20t)+10.8\sin(20x_1(t)x_2(t))+2.2\sin(100x_1(t))\phi_2(t-T) \\ \dot{x}_3(t) &= a(x)+u(t)+\cos(30t)+1.9\sin(50x_1(t)x_3(t)) \\ &\quad +3.1\sin(200x_1(t)x_2(t))\phi_3(t-T)y(t), \end{aligned} \tag{26}$$

where $a(x) = -\exp\left(-\frac{\|x_1-1\|^2}{25}\right)+0.01\sin(0.01x_2)$, $\phi_i(t-T) = \begin{cases} 0, & t<T \\ 1-e^{-\Delta_i(t-T)}, & t \ge T \end{cases}$ $(i=1,2,3)$, $T=5$ s, $\Delta_1=0.07$, $\Delta_2=0.06$, $\Delta_3=0.03$, $G_1=0.1$, $G_2=0.05$, $G_3=1$, the input vector of the NN is $[\hat{x}_1\ \hat{x}_2]$ and the NN structure is 2-7-1. In the network design, taking $c_1=\frac{1}{3}[-3\ -2\ -1\ 0\ 1\ 2\ 3]$, $c_2=\frac{2}{3}[-3\ -2\ -1\ 0\ 1\ 2\ 3]$ and $b=10$, and the initial NN weights are 0. Constructing $V_1=\frac{1}{2}e_3^2+\frac{1}{2}\tilde{W}_3^2$ and choosing $N_1=2$, it follows that $q_1=2$, $\nu_1=3$, $\ell_1=0.5$. Choose $\zeta_2=1.5$, $\zeta_4=0.5$ to satisfy $\nu_1-\frac{1}{\zeta_2}-\frac{1}{\zeta_4}>0$, and set $\zeta_1=10.5$, $\zeta_3=5$, $\zeta_5=15$, $\zeta_6=10$, $z_1=0.1$. Choose $\zeta_7=10$ to satisfy $\bar{\nu}_2-\frac{1}{2\zeta_7}>0$, set $\zeta_8=0.2$, $\zeta_9=15$, $\zeta_{10}=5$, $z_2=80$, and the NN-based observers can be built by Theorem 3.1 and Corollary 3.1. The initial conditions are $x(0)=[0.5\ 0.2\ 0.1]$ and $\hat{x}(0)=[0.1\ 0.5\ 0.2]$, and the input is $20\sin(5t)$.

Calculating $\bar{\xi}=\sqrt{\frac{\bar{G}_3}{z_2}}$, thus the time-varying thresholds can be achieved. The results of the simulation are illustrated in Figure 1 and Figure 2. The residual and time-varying thresholds obtained from Theorem 3.1 are given in Figure 1. From Figure 1, it is shown that the residual is over the lower threshold $-\varphi(t)P\left(\bar{\xi}\right)$, which means that the presented FD method can successfully discover faults after $T>7$ s. To prove the advantages of the method presented in Theorem 3.1, Figure 2 illustrates the tracks of signals obtained from Corollary 3.1. While the constant thresholds $\sqrt{\frac{\bar{G}_3}{z_2}}$ and $-\sqrt{\frac{\bar{G}_3}{z_2}}$ in Corollary 3.1 can detect faults after $T>12$ s, the false alarms are released during transients caused by overshoot before faults occur. The time-varying thresholds in Figure 1 can detect faults faster than the results in Figure 2. Therefore, it can be concluded that the fault detection observer based on time-varying threshold designed by Theorem 3.1 in the same nonlinear system can not only reduce the false alarm caused by overshoot in the transient process, but also reduce the false alarm in the steady-state process and detect the faults as early as possible.

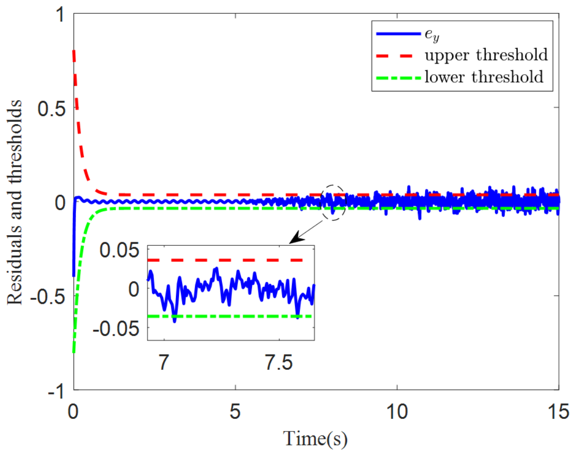

FIGURE 1. The signals yielded by Theorem 3.1

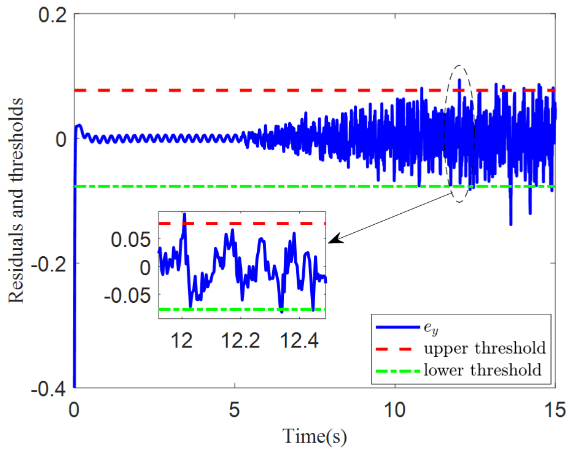

FIGURE 2. The signals yielded by Corollary 3.1

5. **Conclusions.** This paper gives an FD observer method based on NNs and recursive algorithms for the unknown nonlinear systems. A residual generator is constructed using the NN and recursive algorithm to ensure the residual is inside the PPB in the absence of faults. The time-varying thresholds based on the PPB can detect faults faster and reduce false alarms resulting from overshoot. The validity of the approach is confirmed by the simulation results. Further research work includes extending the proposed method to the fault-tolerant control problem of unknown nonlinear systems.

Acknowledgments. This work was supported in part by the Liaoning Natural Science Foundation (Grant Nos. 2022-MS-274, 2023-MS-219), the Shenyang Young and Middle-aged Scientific and Technological Innovation Talents Support Program (RC230346).

REFERENCES

[1] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Springer Science & Business Media, 2012.

[2] Y. L. Xia, A review of the development of artificial neural networks, *Computer Knowledge and Technology*, vol.15, no.20, pp.227-229, 2019.

[3] D. Zhao, Z. Wang and Y. Chen, Proportional-integral observer design for multidelayed sensor-saturated recurrent neural networks: A dynamic event-triggered protocol, *IEEE Transactions on Cybernetics*, vol.50, no.11, pp.4619-4632, 2020.

[4] F. Shen, X. Wang and H. Li, Adaptive output-feedback control for a class of nonlinear systems based on optimized backstepping technique, *International Journal of Adaptive Control and Signal Processing*, vol.5, 36, 2020.

[5] H. Lin, B. Zhao and D. Liu, Data-based fault tolerant control for affine nonlinear systems through particle swarm optimized neural networks, *IEEE/CAA Journal of Automatica Sinica*, vol.7, no.4, pp.954-964, 2020.
[6] F. L. Geng, S. Li and X. X. Huang, Research on fault diagnosis and fault tolerant control of spacecraft attitude control system based on deep neural network, *China Space Science and Technology*, vol.40, no.6, pp.1-12, 2020.
[7] Z. Yang, P. Baraldi and E. Zio, A method for fault detection in multi-component systems based on sparse autoencoder-based deep neural networks, *Reliability Engineering & System Safety*, vol.220, 108278, 2022.
[8] X. Y. Liu, Fault detection of nonlinear system parameters based on adaptive interval model, *Control Engineering*, vol.27, no.9, pp.1626-1635, 2020.
[9] Y. D. Li, Q. L. Hu and X. D. Shao, Neural network-based fault diagnosis for spacecraft with single-gimbal control moment gyros, *Chinese Journal of Aeronautics*, vol.35, no.7, pp.261-273, 2022.
[10] H. Wang and G. H. Yang, Fault detection approaches for fuzzy large-scale systems with unknown membership functions, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.50, no.9, pp.3333-3343, 2018.
[11] Z. H. Zhang and G. H. Yang, Interval observer-based fault isolation for discrete-time fuzzy interconnected systems with unknown interconnections, *IEEE Transactions on Cybernetics*, vol.47, no.9, pp.2413-2424, 2017.
[12] X. Wang, Z. Fei and J. Qiu, Zonotopic fault detection for fuzzy systems with event-triggered transmission, *IEEE Transactions on Fuzzy Systems*, vol.29, no.12, pp.3734-3742, 2020.
[13] F. Boem, S. Riverso and G. Ferrari-Trecate, Plug-and-play fault detection and isolation for large-scale nonlinear systems with stochastic uncertainties, *IEEE Transactions on Automatic Control*, vol.64, no.1, pp.4-19, 2018.
[14] Y. J. Liu, Q. Zeng and S. Tong, Actuator failure compensation-based adaptive control of active suspension systems with prescribed performance, *IEEE Transactions on Industrial Electronics*, vol.67, no.8, pp.7044-7053, 2019.
[15] W. M. Zhang, Prescribed performance based model-free adaptive sliding mode constrained control for a class of nonlinear systems, *Information Sciences*, vol.544, pp.97-116, 2021.
[16] Z. H. Zhang and G. H. Yang, Time-varying threshold-based fault detection for a class of uncertain nonlinear systems in strict-feedback form, *IET Control Theory & Applications*, vol.10, no.17, pp.2149-2159, 2016.
[17] Y. Zeng, T. Chen and C. Wang, Fault diagnosis for a class of nonlinear uncertain systems using deterministic learning approach, *Journal of the Franklin Institute*, vol.360, no.8, pp.5609-5633, 2023.